Note Title 7/11/2006

30% class participate go into questions asked in class for retention of concepts.

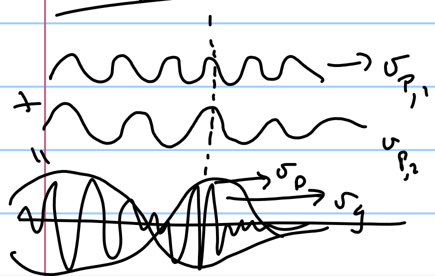
← at fixed location ← change of $\vec{A}$ due to motion thru space

$$\frac{d\vec{A}}{dt} = \frac{\partial \vec{A}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{A}$$

represents the time rate of change of $\vec{A}$ at the location of the moving particle.

$$\left.\frac{\partial r}{\partial t}\right|_{\text{fixed } x,y,z} \qquad \vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

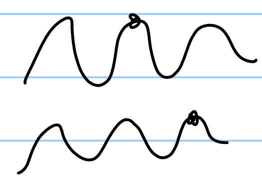
Group vel.




$\varphi = kx - \omega t$

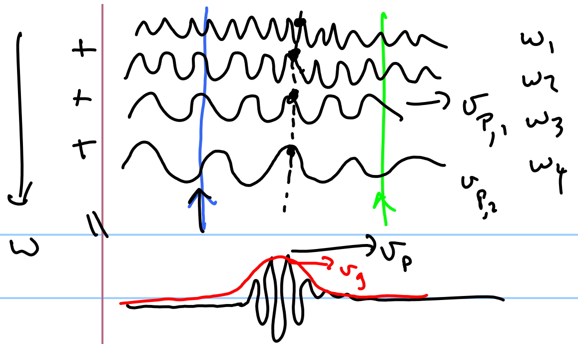
$\delta\varphi = 0 \quad v_{phase} = \frac{\omega}{k}$

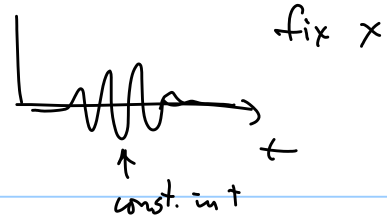
$2\pi\,\text{rad} = \varphi$



$t=0$

wave crest




max all wave add const.

$\varphi(\omega)$ $\qquad \frac{\partial \varphi}{\partial \omega} = 0$ for const. interfer $\varphi$


disp relation $\omega(k)$ or $k(\omega)$ $\qquad \frac{\partial \varphi}{\partial \omega} > 0 \qquad \frac{\partial \varphi}{\partial \omega} < 0$

$$\frac{\partial}{\partial \omega}\left(kx - \omega t\right) = x\frac{dk}{d\omega} - t = 0 \qquad x = \frac{d\omega}{dk}t \qquad x = v_g t$$

say constant

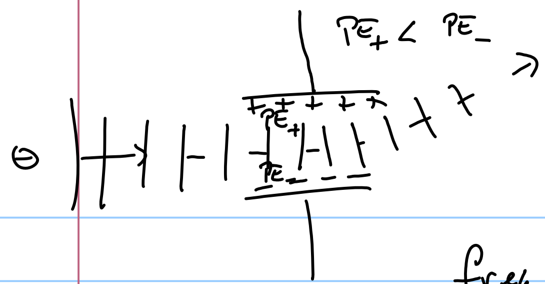
Ex: $\quad E = \frac{p^2}{2m} \qquad p = \hbar k = mv$

$$\boxed{\cancel{\hbar}\omega = \frac{\hbar^{\cancel{2}} k^2}{2m}}$$

$$v_p = \frac{\omega}{k} = \frac{\frac{\hbar k^2}{2m}}{k} = \frac{\hbar k}{2m} = \frac{\cancel{m} v_g}{2\cancel{m}}$$

$$v_p = \frac{v_g}{2}$$

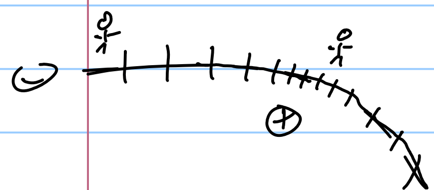
$$v_g = \frac{d\omega}{dk} = \frac{\hbar k}{m} = \frac{mv}{m} = v$$

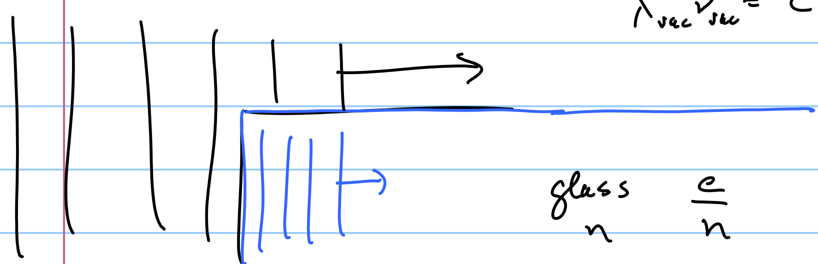



$E_{tot} = \hbar\omega$ $\quad$ $p = \hbar k = \frac{h}{\lambda}$

$E_{tot} = \underbrace{\frac{p^2}{2m}}_{KE} + \underbrace{qV}_{PE}$

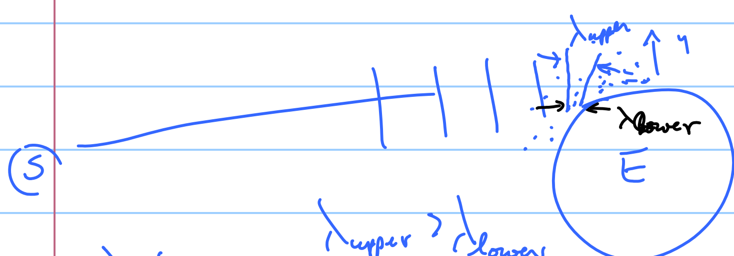
freq wave inside $\qquad$ freq wave outside





$\lambda_{vac}\nu_{vac} = c$

glass $n$ $\qquad$ $\frac{c}{n}$ $\qquad$ $\frac{\lambda}{n}\nu_{vac} = \frac{c}{n}$




$n(y)$

$\lambda_{upper} > \lambda_{lower}$

$\frac{\lambda_{vac}}{n}$

$n(\lambda)$ so $\frac{c}{n(\lambda)}$

Klein-Gordon eqn.

$$\frac{\partial^2 \psi(x,t)}{\partial t^2} = c^2 \frac{\partial^2 \psi(x,t)}{\partial x^2} - \frac{(mc^2)^2}{\hbar^2}\psi(x,t)$$

$$\psi(x,t) = \psi_0 \, e^{i(kx-\omega t)}$$

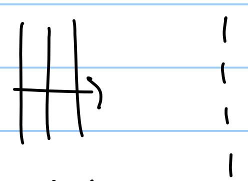
$p = \hbar k$

$E = \hbar \omega$

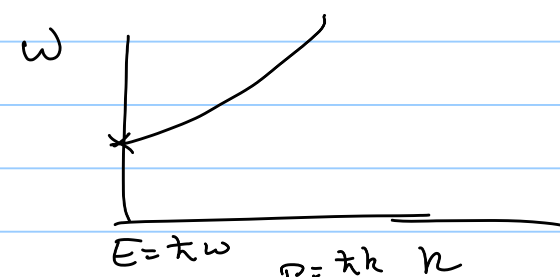
$$\hbar^2\omega^2 = \hbar^2 k^2 c^2 + (mc^2)^2$$

$$E^2 = p^2c^2 + m^2c^4$$

energy mom relation for a relativistic particle

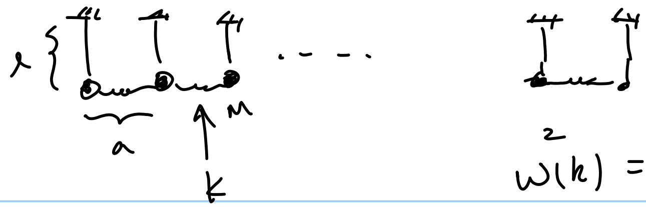


$v \simeq c$

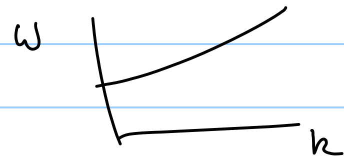



$$\omega = \sqrt{k^2c^2 + (mc^2)^2/\hbar^2}$$

$$v_g = \frac{d\omega}{dk} \qquad v_p = \frac{\omega}{k} = \frac{E/\hbar}{p/\hbar} = \frac{E}{p} = \frac{\dfrac{mc^2}{\sqrt{1-v_g^2/c^2}}}{\dfrac{mv_g}{\sqrt{1-v_g^2/c^2}}} = \frac{c^2}{v_g}$$



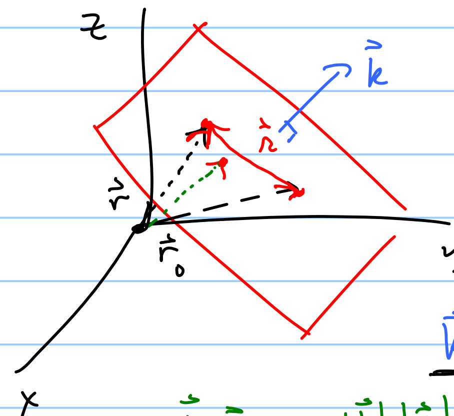

$$\omega^2(k) = \frac{g}{l} + \frac{4K}{M}\sin^2\left(\frac{ka}{2}\right)$$




$$\varphi = \vec{k}\cdot\vec{r} - \omega t$$

fix $t = 0$

eqn for plane in terms of $\vec{k} = \frac{2\pi}{\lambda}\hat{n}$




$\vec{k}\cdot\vec{r}' = 0$

$\vec{r}' = \vec{r} - \vec{r}_0$

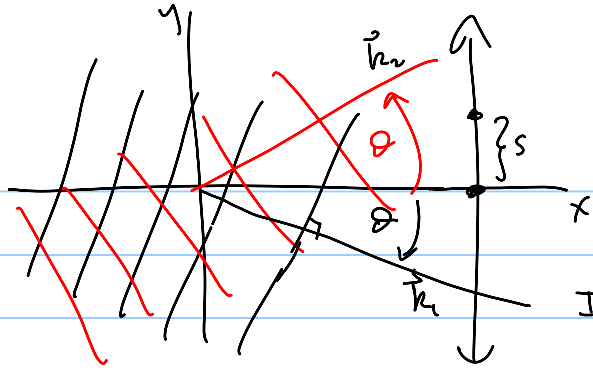
$\vec{k}\cdot\vec{r} = \vec{k}\cdot\vec{r}_0 = \vec{k}\cdot\vec{r}_1$

$\vec{k}\cdot\vec{r}_1 = |\vec{k}||\vec{r}_1|\cos(0) = \frac{2\pi}{\lambda}|\vec{r}_1| =$

⊥ to plane

$\frac{|\vec{r}_1|}{\lambda}$ = # wave crests between origin & plane

$\vec{k}\cdot\vec{r}$ = constant yields points of plane that are ⊥ to $\vec{k}$




$\varphi = \vec{k}\cdot\vec{r} - \omega t$

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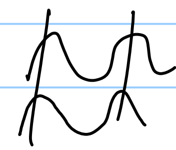
$\Delta\varphi = 0, 2\pi$

If $\Delta\varphi = 0$ construction int.

$\cos\varphi_1 + \cos\varphi_2$



$\varphi_1 = \vec{k}_1\cdot\vec{r} - \omega t$ $\qquad$ $\varphi_2 = \vec{k}_2\cdot\vec{r} - \omega t$

$\vec{r} = x\hat{x} + y\hat{y}$ $\qquad$ $\vec{k}_1 = \frac{2\pi}{\lambda}(\cos\theta\,\hat{x} - \sin\theta\,\hat{y})$

$\Delta\varphi$

$\varphi_1 = \vec{k}_1\cdot\vec{r} = \frac{2\pi}{\lambda}(\cos\theta\,\hat{x} - \sin\theta\,\hat{y})\cdot(x\hat{x} + y\hat{y})$

$\varphi_2 = \vec{k}_2\cdot\vec{r} = \frac{2\pi}{\lambda}(\cos\theta\,\hat{x} + \sin\theta\,\hat{y})\cdot(x\hat{x} + y\hat{y})$

$-\frac{2\pi}{\lambda}(2)\sin\theta\, y = \Delta\varphi$ $\qquad$ solve for $y$ $\Rightarrow$ $\Delta\varphi = 2\pi$