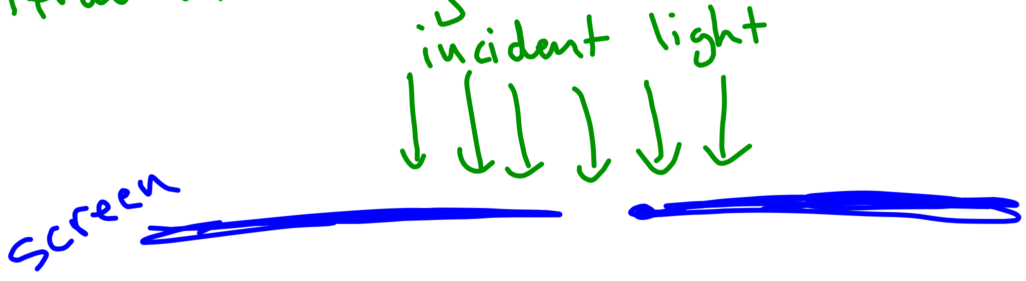
Today: Integral equations: Green's thm, diffraction theory.

Tomorrow: complete sets of functions, how to use mode profiles.

Diffraction Theory:



what happens out here?

Buzz words: Kirchoff integrals
Fresnel Diffraction
Fraunhoffer Diffraction
Diffraction.

Theory:

Green's Functions: Response of some system to "an impulse"

Mathematically, if you have a diff eq. where $f(y, y', y'' \dots) = f_{forcing}(x,t)$

e.g. $\frac{\partial^2 y}{\partial t^2} + k^2 y = f(t)$ {SHO with ext. force $f(t)$}

Green's function for this obeys

$$\frac{\partial^2 G}{\partial t^2} + k^2 G = \delta(t-t')$$

Dirac Delta Function

Result is that you can find

$$y(t) = \int_{-\infty}^{t} G(t,t') f(t')\, dt'$$

In E&M, it turns out for our wave equation $\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$. If we assume

$\vec{E} = \vec{E}(\vec{r}) e^{-i\omega t}$ {time dependence is all at same frequency}

$$\Rightarrow \nabla^2 \vec{E} + \underbrace{\frac{\omega^2}{c^2}}_{k^2} \vec{E} = 0 \quad \text{\{Helmholtz Eqn.\}}$$

$$\Rightarrow \nabla^2 \vec{E} + k^2 \vec{E} = 0$$

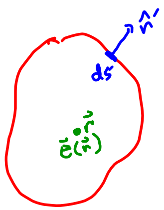
$$(\nabla^2 + k^2)\vec{E} = 0$$

It turns out $G(\vec{r}, \vec{r}') = \dfrac{e^{ik|\vec{r}-\vec{r}'|}}{4\pi|\vec{r}-\vec{r}'|}$

$$\vec{E}(\vec{r}) = \oint_s \vec{E}\left(\frac{\partial G}{\partial n}\right) - G\left(\frac{\partial \vec{E}}{\partial n}\right) ds'$$

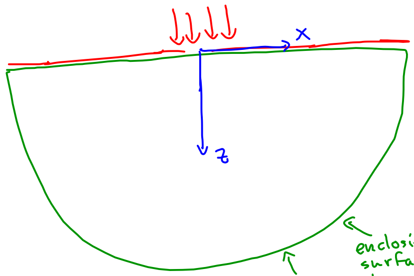
Green's Thm for EM waves.

$\frac{\partial}{\partial n} = \hat{n} \cdot \vec{\nabla}$



Everything so far is true in general.

Ok. so how is this applied in diffraction theory.



enclosing surface

take limit as radius $\to \infty$ and $\int_{\text{semi circle}} = \emptyset$.

Pretty much always true.

So now our $\oint$ becomes just the integral of the field (E_y) and normal derivative ($-\frac{\partial E_y}{\partial z}$)

$$\Rightarrow \vec{E}(\vec{r}) = \int_{-\infty}^{\infty} \vec{E}\frac{\partial G}{\partial z} + G\frac{\partial \vec{E}}{\partial z}\,dx$$; $\vec{E}$ is the field just below the screen.

You can recast this in different ways, and one of the most common is

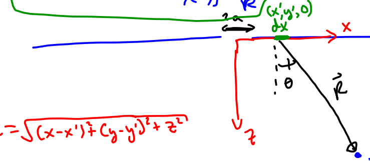
So the most ubiquitous version is the Kirchoff integral:

$$\vec{E}(x,y,z) = \frac{k}{2\pi i}\iint \frac{e^{ikR}}{R}\left(1+\frac{i}{kR}\right)\cos\theta\, E(x',y',0)\,dx'dy'$$

Wikipedia:

$$\vec{E}(x,y,z) = \frac{-i}{\lambda}\iint \frac{e^{ikR}}{R}\cos\theta\, E(x',y',0)\,dx'dy'$$

$R = |\vec{r}-\vec{r}\,'|$



$R = \sqrt{(x-x')^2+(y-y')^2+z^2}$

$\vec{r} = x\hat{x}+y\hat{y}+z\hat{z}$
(x,y,z)

$\theta = \frac{z}{R}$

Example: single slit: $E_y(x') = \begin{cases} 0, & |x|>a \\ E_0, & |x'|<a \end{cases}$ (y-polarization)

{assume no y-dep}

$$E_y(x,z) = \frac{-i}{\lambda}\int_{-\infty}^{\infty} E_y(x')\frac{e^{ikR}}{R}\frac{z}{R}\,dx'$$

$$= \frac{-i}{\lambda}\int_{-a}^{a} E_0\frac{e^{ikR}}{R}\frac{z}{R}\,dx'$$

Tomorrow: