Homework 4, due Tuesday 4/11

Feel free to use Mathematica.

1. Let $\psi_n(x) = c_n e^{-\alpha^2 x^2/2} H_n(\alpha x)$ where $\alpha^2 = \frac{m\omega}{\hbar}$. See equation 4.16 in the book. Show that ψ_0, ψ_1 and ψ_2 are orthogonal to one another.

2. Evaluate $\langle x \rangle$, $\langle p \rangle$, $\langle x^2 \rangle$ and $\langle p^2 \rangle$ for the $n = 1$ harmonic oscillator state.

3. Consider a classical oscillator whose energy is $\hbar\omega/2$. Find the limit of its motion (A in the notes). What is the probability that a quantum particle with the same energy will be found more than twice this far from the origin?

4. Apply separation of variables to the Schrodinger equation for a 3D potential well defined by: $V(x, y, z) = V(x)+V(y)+V(z)$ where $V(x_i) = 0$ if $0 \leq x_i \leq L_i$ and infinity otherwise. Derive the expression for the wavefunctions $\psi_{n_1,n_2,n_3}(\mathbf{r})$ and E_{n_1,n_2,n_3}.

5. Suppose that $\psi(x, 0) = e^{ip_0 x/\hbar}\psi_0(x)$. Find the wavefunction as a function of time. What is the average energy of the oscillator?