Today: Complete sets of functions

Remember from Quantum, in some generalized space, you can write any function $|f\rangle = \sum \underbrace{\langle g_n|f\rangle}_{\text{overlap of } f \text{ onto } |g_n\rangle} |g_n\rangle$

$|g_n\rangle$ are the ortho-normal basis functions for your space.

For EM waves in free space your orthonormal set are basically $e^{i(\vec{k}\cdot\vec{r}-\omega t)}$ but the sum is actually an integral because in free space all $\vec{k}$'s are allowed.

Any wave can be written as

$$\tilde{f}(\vec{r},t) = \underset{\text{all } \vec{k}}{\iiint} \underset{\omega}{\int} \tilde{A}(\vec{k},\omega)\, e^{i(\vec{k}\cdot\vec{r}-\omega t)}\, d^3k\, d\omega$$

Situations with waveguides is a little different, and more like what you did in Quantum.

We've already solved for some bound modes of waveguides.

{let's just work w/ fixed ω}

$$f(\vec{r},t) = \sum_{\substack{\text{over bound } \vec{k}\text{'s} \\ (\text{over } m)}} \tilde{A}(\vec{k})\, f_{\text{bound},m}(\vec{r},\vec{k})\, e^{-i\omega t} + \underbrace{\int \tilde{B}(\vec{k})\, f_{\text{radiative}}(\vec{r},\vec{k})\, d\vec{k}}_{\text{we aren't going to work with this.}}$$

We'll look at finding how much of an incident field goes into each of the modes of a waveguide.

In Quantum, in position space (x)

$$\langle f|g\rangle = ? = \int_{\substack{\text{all} \\ \text{space}}} g f^* \, dx$$

$\Rightarrow$ How to find how much of an incident wave $u_0(x)$ goes into each bound mode, this is how you do it.

1) Find β's for all bound modes.

2) Use those to find $u_{b,i}(x)$ $\quad u_{b1}$ = first bound mode.

3) Normalize $u_{b,i}(x)$ using

$$u_{b,i}(x) \Rightarrow \frac{u_{b,i}(x)}{\sqrt{\int_{-\infty}^{\infty} u_{b,i}(x)\, u_{b,i}^*(x)\, dx}}$$

4) Take overlap $\langle u_{b,i}|u_0\rangle$

$$K_i = \int_{-\infty}^{\infty} u_0\, u_{b,i}^*\, dx$$

$K_i/|u_0|^2$ is the fraction of u_0 to go into the i^{th} mode.