11

pulse characterization

NL index

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Correlation vs convolution

- Correlation:

$$g_{AC}(\tau) = \int_{-\infty}^{\infty} f_1(t) f_2^*(t-\tau)\,dt$$

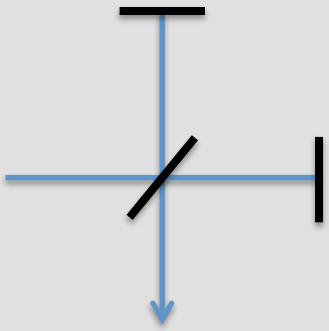
- Convolution:

$$g(\tau) = f_1 \otimes f_2 = \int_{-\infty}^{\infty} f_1(t) f_2(\tau - t)\,dt$$

- Autocorrelation and autoconvolution are operations on the same pulse.
- Michelson interferometer:
  - Time ordering of the pulse cannot change
  - Detector measures time-integrated intensity

$$e_{sig} \propto \int \left| E_1(t) + E_2(t-\tau) \right|^2 dt$$

$$\tau = \frac{L_1 - L_2}{c}$$



Graphical approach to convolution

Input functions

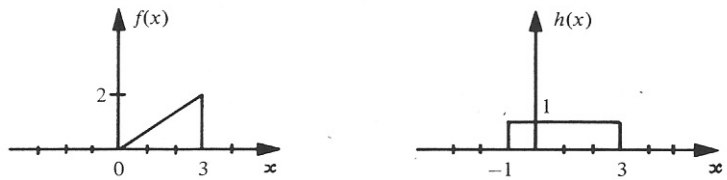


Figure 6-1 Functions used to illustrate convolution by graphical methods.

output

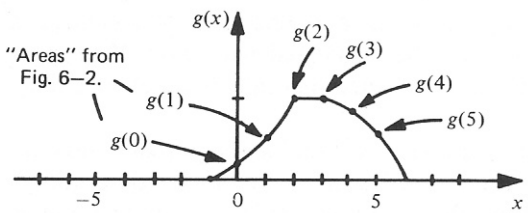


Figure 6-3 Resulting convolution of functions shown in Fig. 6-1.

Graphical view

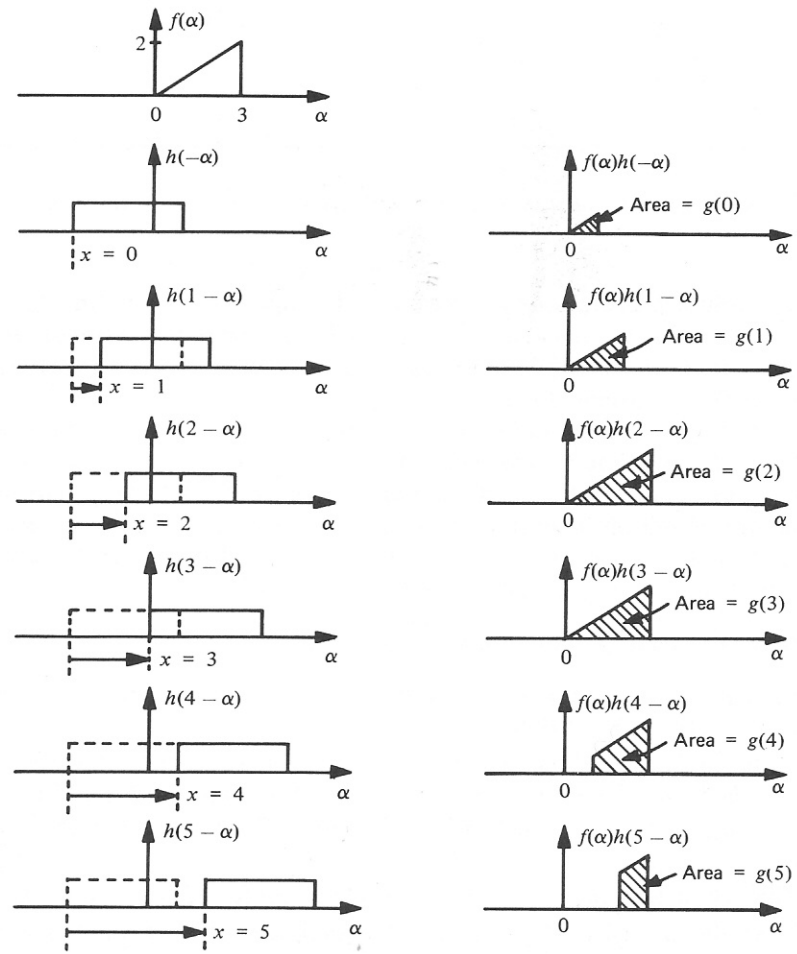


Figure 6-2 Graphical method for convolving functions of Fig. 6-1.

Smoothing effects

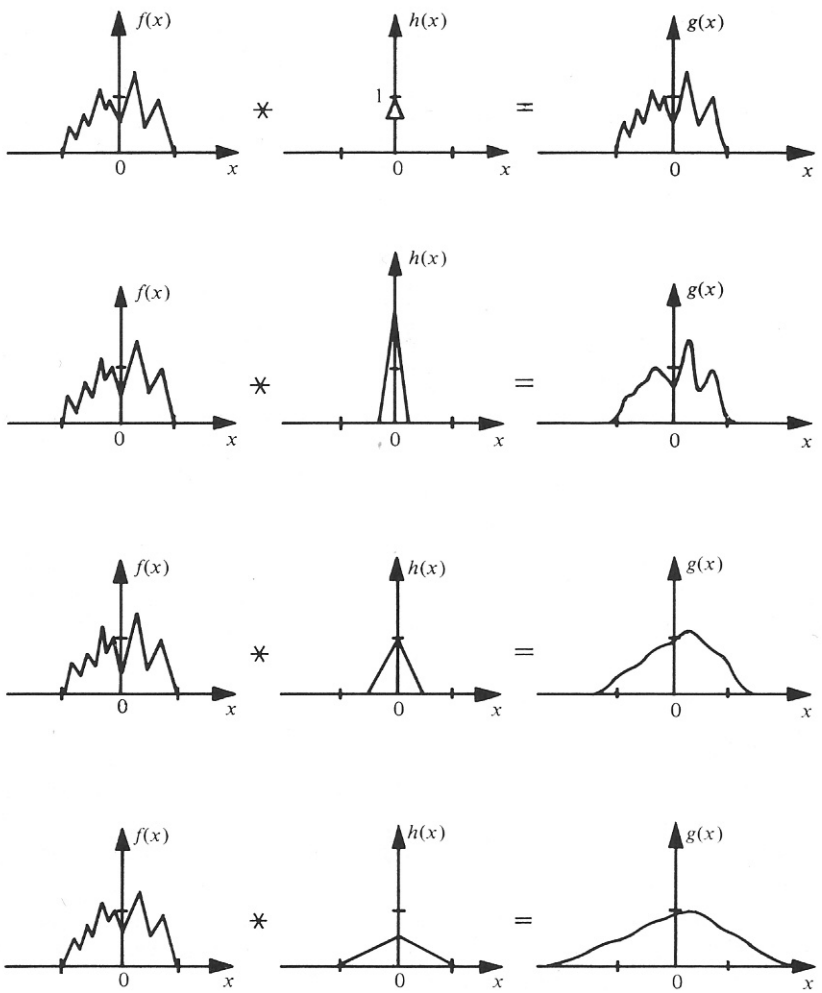


Figure 6-5 Smoothing effects of convolution.

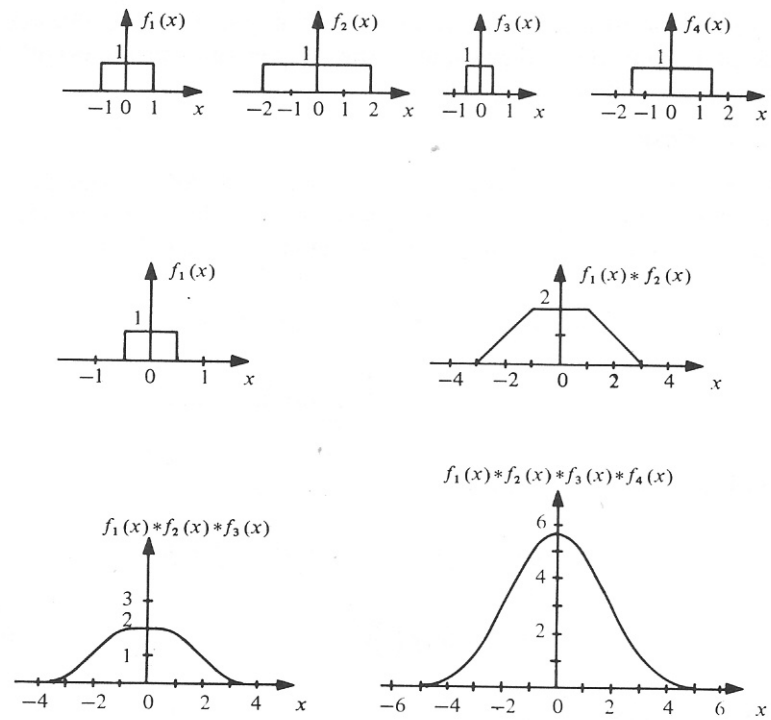


Figure 6-7 Repeated convolution of four rectangle functions.

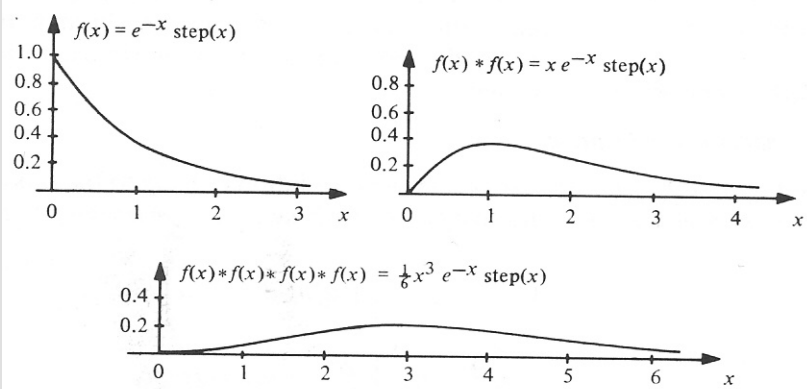


Figure 6-8 Repeated convolution of the function $\exp\{-x\}\text{step}(x)$.

Convolution theorem

$$FT^{-1}\left\{F(\omega)G(\omega)\right\} = FT^{-1}\left\{\left[\int_{-\infty}^{\infty} f(t')e^{+i\omega t'}\,dt'\right]\left[\int_{-\infty}^{\infty} g(t'')e^{+i\omega t''}\,dt''\right]\right\}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\int_{-\infty}^{\infty} dt' f(t')\int_{-\infty}^{\infty} dt'' g(t'')e^{+i\omega(t'+t'')}$$

$$= \int_{-\infty}^{\infty} dt' f(t')\int_{-\infty}^{\infty} dt'' g(t'')\underbrace{\frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\; e^{+i\omega(t'+t'')}e^{-i\omega t}}_{\delta(t-(t'+t''))}$$

$$= \int_{-\infty}^{\infty} dt' f(t')\int_{-\infty}^{\infty} dt'' g(t'')\delta\left(t''-(t-t')\right) = \int_{-\infty}^{\infty} dt' f(t')g(t-t') = f\otimes g$$

$$\boxed{FT\left\{f(t)\otimes g(t)\right\} = F(\omega)G(\omega)}$$

$$\boxed{FT\left\{f(t)g(t)\right\} = \frac{1}{2\pi}F(\omega)\otimes G(\omega)}$$

Autocorrelation theorem

$$FT^{-1}\left\{F(\omega)G^*(\omega)\right\} = FT^{-1}\left\{\left[\int_{-\infty}^{\infty} f(t')e^{+i\omega t'}\,dt'\right]\left[\int_{-\infty}^{\infty} g^*(t'')e^{-i\omega t''}\,dt''\right]\right\}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\int_{-\infty}^{\infty} dt' f(t')\int_{-\infty}^{\infty} dt'' g^*(t'')e^{+i\omega(t'-t'')}$$

$$= \int_{-\infty}^{\infty} dt' f(t')\int_{-\infty}^{\infty} dt'' g^*(t'')\underbrace{\frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\; e^{+i\omega(t'-t'')}e^{-i\omega t}}_{\delta(t-(t'-t''))}$$

$$= \int_{-\infty}^{\infty} dt' f(t')\int_{-\infty}^{\infty} dt'' g^*(t'')\delta\left(t''-(t'-t)\right) = \int_{-\infty}^{\infty} dt' f(t')g^*(t'-t)$$

If G = F,

$$FT\left\{\int_{-\infty}^{\infty} dt' f(t')f^*(t'-t)\right\} = F(\omega)F^*(\omega) = \left|F(\omega)\right|^2$$

Measuring pulse duration: field autocorrelation?

- The output of a linear Michelson interferometer is connected to the autocorrelation of the field

$$e_{sig} \propto \int \left| E_1(t) + E_2(t-\tau) \right|^2 dt$$

$$e_{sig} \propto \int \left(\left| E_1(t) \right|^2 + \left| E_2(t-\tau) \right|^2 + 2\,\mathrm{Re}\left(E_1(t) E_2^*(t-\tau) \right) \right) dt$$

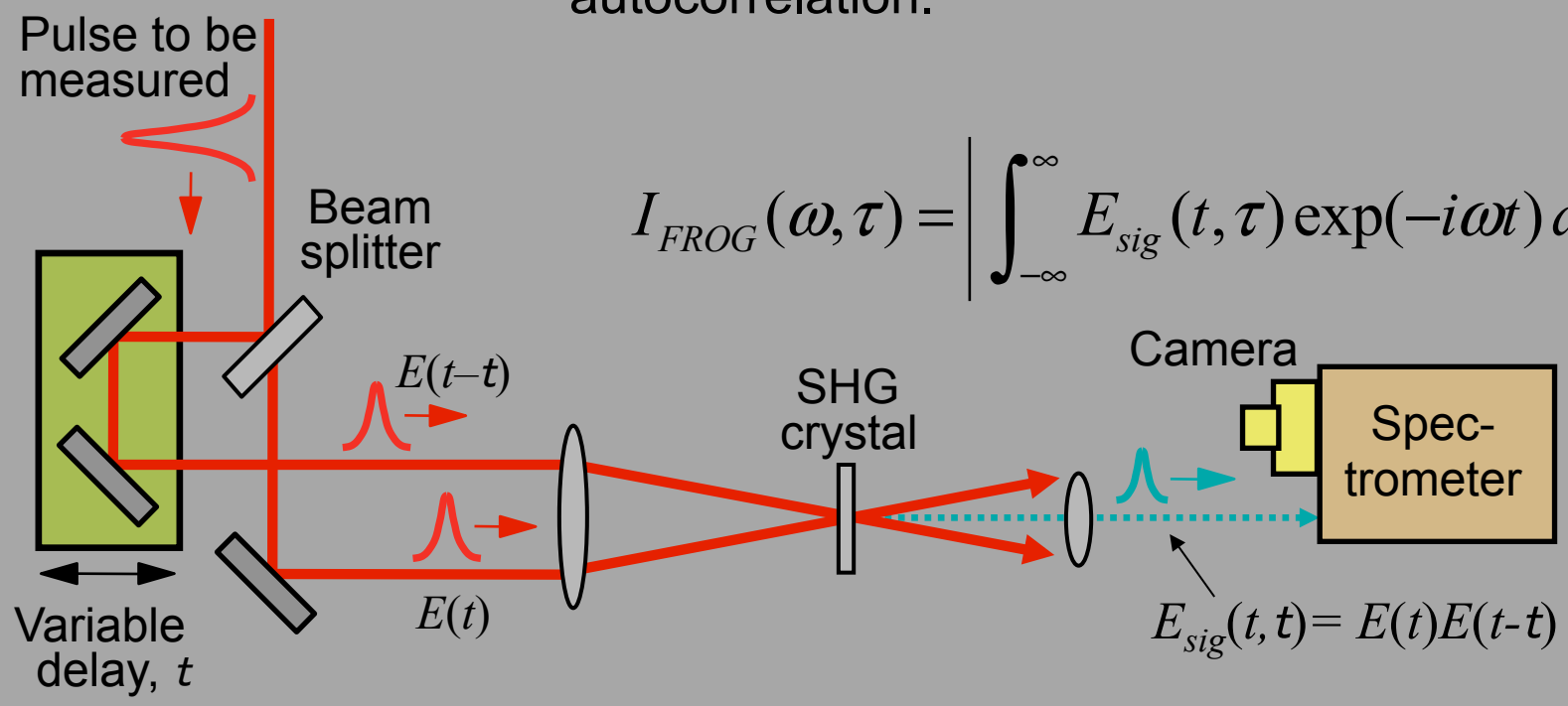
$$e_{sig} \propto e_1 + e_2 + 2\,\mathrm{Re}\left(g_{AC}(\tau) \right)$$

  - If beamsplitter is 50/50, then e1=e2
- What can we learn by measuring the autocorrelation of the field?
  - We get the power spectrum
  - phase info is gone, so no pulse duration measurement

Second-harmonic-generation FROG

SHG FROG is simply a spectrally resolved autocorrelation.

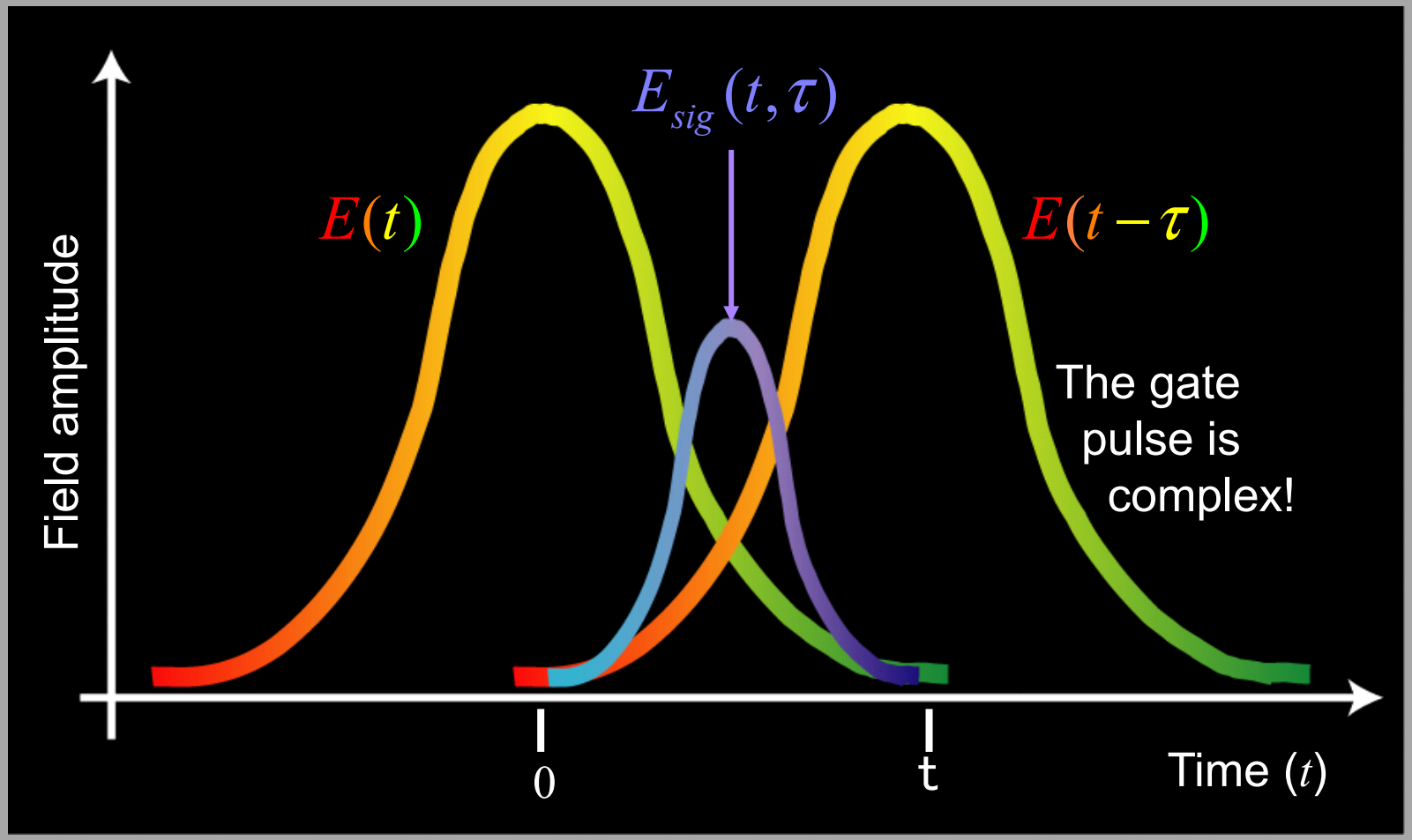
$$I_{FROG}(\omega,\tau) = \left| \int_{-\infty}^{\infty} E_{sig}(t,\tau)\exp(-i\omega t)\,dt \right|^2$$



SHG FROG is the most sensitive version of FROG.

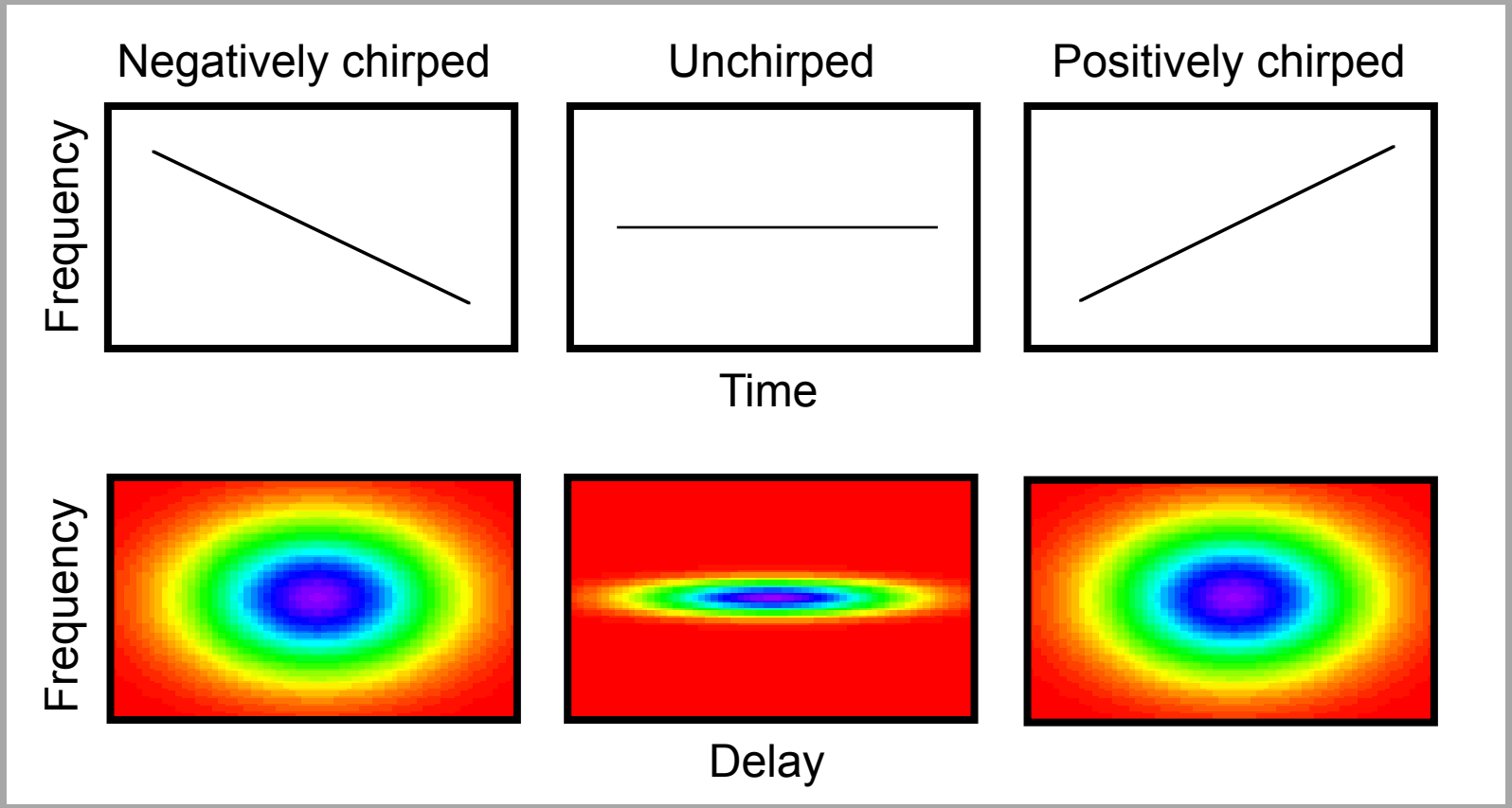
SHG FROG

$$E_{sig}(t,\tau) \propto E(t)E(t-\tau)$$



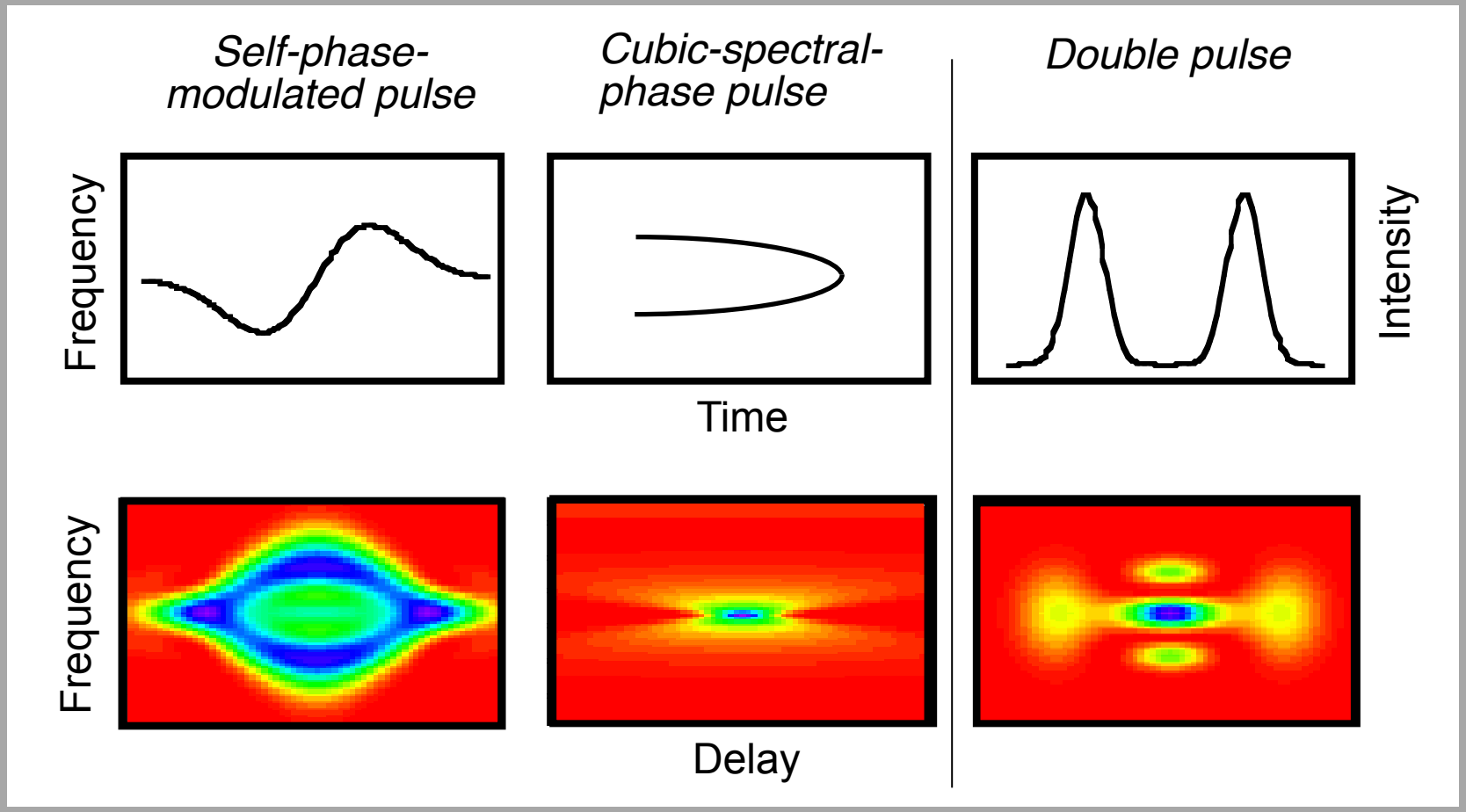
SHG FROG is also a spectrogram, but its interpretation is more complex.

SHG FROG traces are symmetrical with respect to delay.



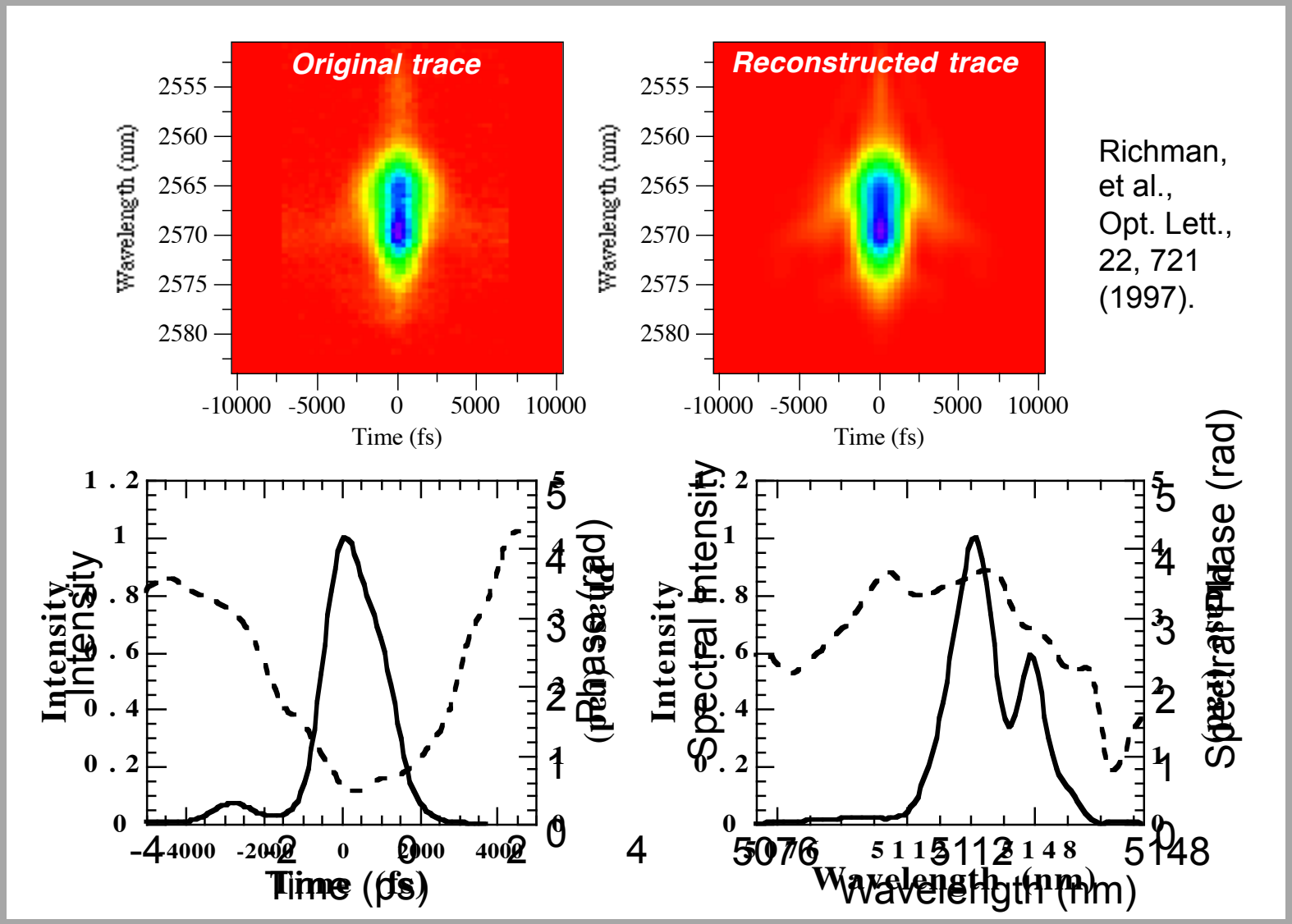
SHG FROG has an ambiguity in the direction of time, but it can be removed.

SHG FROG traces for complex pulses



SHG FROG traces are symmetrized PG FROG traces.

SHG FROG Measurements of a Free-Electron Laser



Richman, et al., Opt. Lett., 22, 721 (1997).

SHG FROG works very well, even in the mid-IR and for difficult sources.

The FROG Marginals

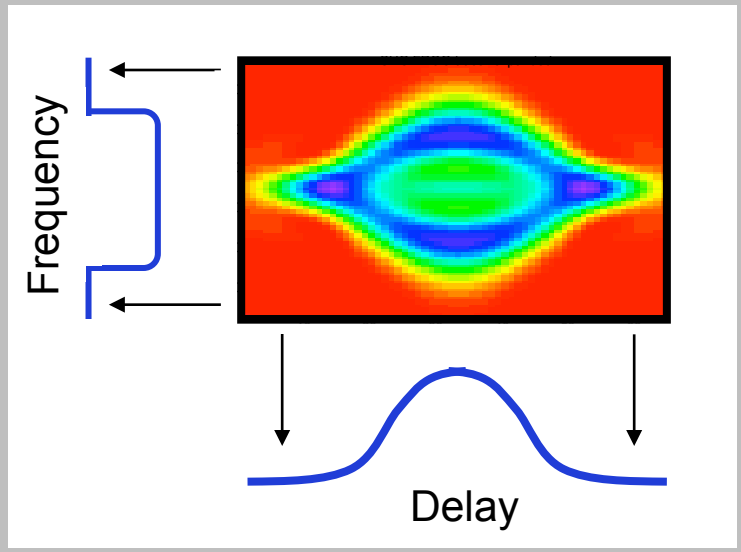
The **delay marginal** is the integral of the FROG trace over all frequencies. It is a function of delay only.

$$M_\tau(\tau) \equiv \int I_{FROG}(\omega,\tau)\, d\omega$$

The **frequency marginal** is the integral of the FROG trace over all delays: It is a function of frequency only.

$$M_\omega(\omega) \equiv \int I_{FROG}(\omega,\tau)\, d\tau$$

The marginals are essential in checking for systematic error.



The SHG FROG marginals can be related to easily measured quantities:

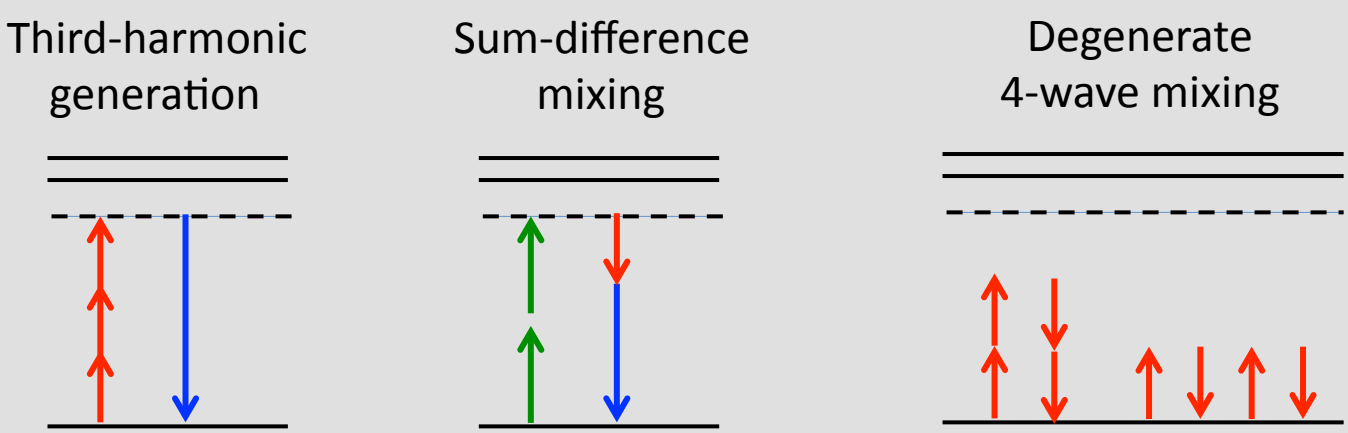
$M_\tau(\tau)$ = The Autocorrelation

$M_\omega(\omega)$ = The Autoconvolution of the Spectrum

DeLong, et al., JQE, 32, 1253 (1996).

Intensity dependent refractive index

- $\chi^{(3)}$ effects lead to four-wave mixing



- DFWM seems like nothing is produced, but:
  - NL refractive index leads to NL phase changes
  - NL ellipse rotation, polarization changes
  - Cross-polarized wave generation
  - Many effects: self-phase mod, self-focusing, solitons, phase conjugation, transient gratings, …

Nonlinear Refractive Index

The refractive index in the presence of linear *and nonlinear* polarization:

$$n = \sqrt{1 + \chi^{(1)} + 3\chi^{(3)} \left| E \right|^2}$$

Now, the usual refractive index (which we'll call n_0) is: $n_0 = \sqrt{1 + \chi^{(1)}}$

So: $$n = \sqrt{n_0^2 + 3\chi^{(3)} \left| E \right|^2} = \sqrt{n_0^2 + 3\chi^{(3)} \frac{I}{2 n_0 \varepsilon_0 c}}$$

Assume that the nonlinear term << n_0:

So: $$n = n_0 \sqrt{1 + \frac{3\chi^{(3)}}{2 n_0^3 \varepsilon_0 c} I} \approx n_0 \left(1 + \frac{3\chi^{(3)}}{4 n_0^3 \varepsilon_0 c} I \right) = n_0 + \frac{3\chi^{(3)}}{4 n_0^2 \varepsilon_0 c} I$$

Usually, we define a "nonlinear refractive index":

$$\boxed{n \approx n_0 + n_2 I} \qquad n_2 = \frac{3\chi^{(3)}}{4 n_0^2 \varepsilon_0 c}$$

See Boyd 4.1 for other definitions.

Mechanisms and time scales for NL index

Table 1. Representative Materials with Values of n_2 and α_2[a]

Material	E_g (eV)	$n_2 \times 10^{-15}$ (cm^2/W)				α^2 (cm/GW)		
		1064 nm	532 nm	355 nm	266 nm	532 nm	355 nm	266 nm
LiF	11.6	0.081	0.061	0.061	0.13	~0	~0	~0
MgF_2	11.3	0.057	0.057	0.066	0.15	~0	~0	~0
BaF_2	9.2	0.14	0.21	0.27	0.31	~0	~0	0.06
NaCl [6]	~8.7	1.8						
SiO_2	~7.8	0.21	0.22	0.24	0.78	~0	~0	0.05
MgO [6]	7.77	0.39						
Al_2O_3	7.3	0.31	0.33	0.37	0.60	~0	~0	0.09
BBO	6.2	0.29	0.55	0.36	0.003	~0	0.01	0.9
KBr	6.0	0.79	1.27			~0		
$CaCO_3$	5.9	0.29	0.29	0.37	1.2		0.018	0.8
$LiNbO_3$	3.9	0.91	8.3			0.38		
KTP	3.8	2.4	2.3			0.1		
ZnS [9]	3.66	6.3				3.4		
Te Glass	~3.6	1.7	9.0			0.62		
ZnSe [9]	2.67	29	−68			5.8		
ZnTe [10]	2.26	120				4.2		
CdTe [10]	1.44	−300				22		
GaAs [10]	1.42	−330				26		
RNglass [11]	~1.4	2.2						

[a]Ordered according to bandgap energy, E_g, or cutoff wavelength, taken from [8] except where noted. The values quoted were obtained by using multiple pulse widths in order to isolate the Kerr response. See the references for details. Blank cells indicate no measurement at this wavelength.

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Electronic (fs)
Molecular orientation (ps)
Electrostriction (ns)
Saturated absorption (10ns)
Thermal (ms)
Photorefractive (varies, slow)

Mechanisms for NL index

1) Electronic polarization : Electronic charge redistribution

2) Molecular orientation : Molecular alignment due to the induced dipole

3) Electrostriction : Density change by optical field

4) Saturated absorption : Intensity-dependent absorption

5) Thermal effect : Temperature change due to the optical field

6) Photorefractive effect : Induced redistribution of electrons and holes ➔

Refractive index change due to the local field inside the medium

TABLE 4.1.1 Typical values of the nonlinear refractive index[a]

Mechanism	n_2 (cm^2/W)	$\chi^{(3)}_{1111}$ (esu)	Response time (sec)
Electronic polarization	10^{-16}	10^{-14}	10^{-15}
Molecular orientation	10^{-14}	10^{-12}	10^{-12}
Electrostriction	10^{-14}	10^{-12}	10^{-9}
Saturated atomic absorption	10^{-10}	10^{-8}	10^{-8}
Thermal effects	10^{-6}	10^{-4}	10^{-3}
Photorefractive effect [b]	(large)	(large)	(intensity-dependent)

[a] For linearly polarized light.

[b] The photorefractive effect often leads to a very strong nonlinear response. This response usually cannot be described in terms of a $\chi^{(3)}$ (or an n_2) nonlinear susceptibility, because the nonlinear polarization does not depend on the applied field strength in the same manner as the other mechanisms listed.