

In order to receive full credit, **SHOW ALL YOUR WORK**. Full credit will be given only if all reasoning and work is provided. Please enclose your final answers in boxes.

1. (10 Points) Define  $M_{2 \times 2}$  as the vector space of all two-by-two matrices with real entries. Let  $H$  be the subset of all real matrices of the form  $\mathbf{A} = \begin{bmatrix} a & 0 \\ c & d \end{bmatrix}$ . Is  $H$  a subspace of  $M_{2 \times 2}$ ? Justify your response.

2. (10 Points) Let  $\lambda$  be an eigenvalue of the invertible matrix  $\mathbf{A}$ . Show that  $\lambda^{-1}$  is an eigenvalue of  $\mathbf{A}^{-1}$ .

3. (10 Points) Given that,

$$\mathbf{A} = \begin{bmatrix} 2 & 0 \\ 1 & 0 \end{bmatrix}.$$

Determine a diagonal decomposition,  $\mathbf{PDP}^{-1}$ , of  $\mathbf{A}$ .

4. (10 Points) Which of the following matrices are diagonalizable?

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

Explain.