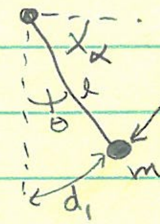
PH 350 Fall 2011 Practice Test 2 solns

1) a) There is only 1 degree of freedom, described by the angle as the pendulum swings back and forth.

b) θ, α, $d_1 = l\theta$ or $d_2 = l\alpha$ would all give you relatively simple lagrangians of only 1 coordinate. Not and! Only 1 coordinate is used.



c) Use θ: $x = l\sin\theta$; $y = -l\cos\theta$

$\dot{x} = l\cos\theta\,\dot{\theta}$; $\dot{y} = +l\sin\theta\,\dot{\theta}$

$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{y}^2 = \frac{1}{2}ml^2\dot{\theta}^2$ $(\sin^2\theta + \cos^2\theta = 1)$.

$U = mgy = -mgl\cos\theta$

$$\rightarrow \boxed{L = \frac{1}{2}ml^2\dot{\theta}^2 + mgl\cos\theta}$$

d) $H = \frac{\partial L}{\partial\dot{\theta}}\dot{\theta} - L = ml^2\dot{\theta}\dot{\theta} - \frac{1}{2}ml^2\dot{\theta}^2 - mgl\cos\theta$

$$\boxed{H = \frac{1}{2}ml^2\dot{\theta}^2 - mgl\cos\theta}$$

H is supposed to be a function of θ, P_θ. On the test, I may require you to do this conversion

$P_\theta = \frac{\partial L}{\partial\dot{\theta}} = ml^2\dot{\theta} \rightarrow \dot{\theta} = \frac{P_\theta}{ml^2}$

$$\rightarrow \boxed{H(\theta, P_\theta) = \frac{P_\theta^2}{2ml^2} - mgl\cos\theta}$$

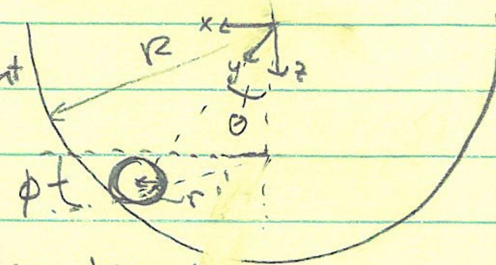
e) Yes, $\frac{dH}{dt} = \frac{\partial H}{\partial t}$ and there is no exp. t-dep.

f) Yes, $x(\theta;t)$; $y(\theta;t)$ don't depend on t.

g) I like (g) too.

2) a) 6 : 3 rotational and 3 translational.

b) An object constrained to any 2-D surface is constrained in one translational direction (it can't come off the surface)
→ 1 translational constraint.



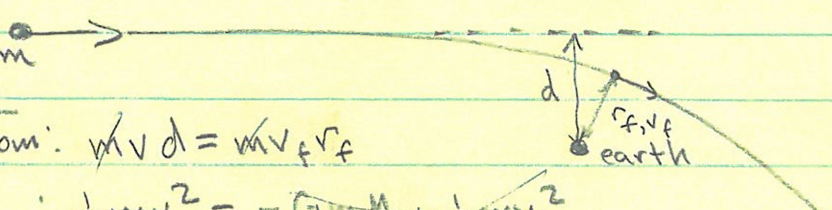
This could be written as the single constraint equation $x_{cm}^2+y_{cm}^2+z_{cm}^2=(R-r)^2$

Rotationally you lose all degrees of freedom because if it's not slipping then $\omega_{\perp \text{ plane of motion}}=\frac{v}{r}$ and the two components of ω in the plane of motion $=\phi$. → 3 constraints.
3 rotational constraints

$\boxed{\text{4 constraints}}$ → $\boxed{\text{2 deg. of freedom}}$

c) See the above picture: I would pick the standard θ, ϕ from spherical coordinates. Anything proportional to those will be identical. If you chose something different, be sure you motivate why you chose it. For instance my coords. give simple transformations: $x=(R-r)\sin\theta\cos\phi$; $y=(R-r)\sin\theta\sin\phi$
$z=(R-r)\cos\theta$

3)



Cons. of ang. mom: $\cancel{m}vd=\cancel{m}v_f r_f$

Note! I find out this is wrong later.

Energy : $\frac{1}{2}mv^2=-\frac{GmM}{r_f^2}\cancel{+\frac{1}{2}mv_f^2}$
($u=0$ @ ∞)

$\rightarrow r_f=\frac{vd}{v_f} \rightarrow \frac{1}{2}mv^2=-\frac{GmM}{v^2d^2/v_f^2}+\frac{1}{2}mv_f^2=\cancel{\left(-\frac{GmM}{v^2d^2}+\frac{1}{2}m\right)v_f^2}$

(see next page)

3 cont'd) $\rightarrow \frac{1}{2}\cancel{m}v^2 = \left(-\frac{G\cancel{m}M}{v^2d^2} + \frac{1}{2}\cancel{m}\right)v_f^2$

$$\rightarrow \cancel{\boxed{v_f^2 = \frac{v^2}{\left(1 - \frac{2GM}{v^2d^2}\right)}}}$$

or $\rightarrow \cancel{\boxed{v_f = \frac{v}{\sqrt{1 - \frac{2GM}{v^2d^2}}}}}$ $\rightarrow r_f = \frac{vd}{v_f} = d\sqrt{1 - \frac{2GM}{v^2d^2}}$

Units: $\frac{GM}{d^2} = [m/s^2] \rightarrow \frac{GM}{v^2d^2} = \left[\frac{1}{m}\right]$ doesn't work. Hmm.

Oops $U \neq -\frac{GmM}{r^2} \rightarrow U = -\frac{GmM}{r}$. Starting over

$\rightarrow \frac{1}{2}mv^2 = -\frac{GmM}{r_f} + \frac{1}{2}mv_f^2$; $r_f = \frac{vd}{v_f}$ {this was still good}

$$\rightarrow \frac{1}{2}mv^2 = -\frac{GmM}{vd/v_f} + \frac{1}{2}mv_f^2 = -\frac{GmM}{vd}v_f + \frac{1}{2}mv_f^2$$

$$\rightarrow v_f^2 - \frac{2GM}{vd}v_f - v^2 = \emptyset$$

$$\rightarrow v_f = \frac{\frac{2GM}{vd} \pm \sqrt{\frac{4G^2M^2}{v^2d^2} + 4v^2}}{2} = \frac{GM}{vd} \pm \sqrt{\frac{G^2M^2}{v^2d^2} + v^2}$$

$$\boxed{v_f = \frac{GM}{vd}\left(1 \pm \sqrt{1 + \frac{v^4d^2}{G^2M^2}}\right); \quad r_f = \frac{vd}{v_f}}$$

Now units:

$\frac{GM}{d^2} = [m/s^2] \rightarrow \frac{G^2M^2}{d^2} = \left[\frac{m^2}{s^4} \cdot m^2\right] \rightarrow \frac{v^2d^2}{G^2M^2} = [1]$ ✓ $\frac{GM}{vd} = \left[\frac{m}{s^2} \cdot m \cdot \frac{s}{m}\right] = \left[\frac{m}{s}\right]$ ✓

+ or −? The $\sqrt{\ }$ is alway > 1 so for $v_f > \emptyset$, it must be (+). $\boxed{\text{Use + root}}$

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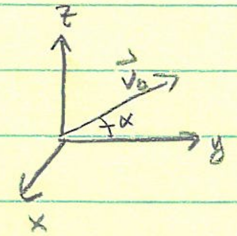
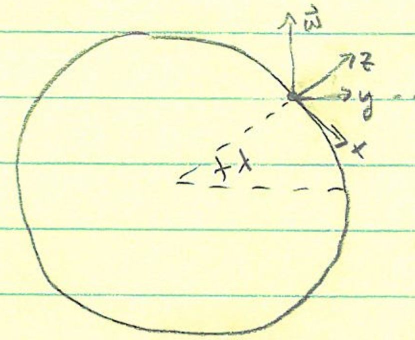
4)

$\vec{a}_r = \vec{g} - 2\,\vec{\omega}\times\vec{v}_r$





$\vec{v}_r \sim v_0\cos\alpha\,\hat{e}_y$
$\quad + (v_0\sin\alpha - gt)\,\hat{e}_z$

$\vec{\omega} = -\omega\cos\lambda\,\hat{e}_x + \omega\sin\lambda\,\hat{e}_z$

$\vec{\omega}\times\vec{v}_r = -\omega v_0\cos\lambda\cos\alpha\,\hat{e}_z + \omega\cos\lambda\,(v_0\sin\alpha - gt)\,\hat{e}_y$
$\quad - \omega v_0\cos\alpha\sin\lambda\,\hat{e}_x$

$\rightarrow \vec{a}_r \sim (-g - \omega v_0\cos\lambda\cos\alpha)\,\hat{e}_z$
$\quad + (2\omega v_0\cos\alpha\sin\lambda)\,\hat{e}_x \; - 2\omega\cos\lambda\,(v_0\sin\alpha - gt))\,\hat{e}_y$

$a_{r,z} \sim -g \quad\rightarrow\quad v_{r,z} = v_0\sin\alpha - gt\,; \quad z(t) = v_0\sin\alpha\,t - \frac{1}{2}gt^2$

$t_f = t(z=\emptyset) = \dfrac{2v_0\sin\alpha}{g}$

$d = x(t_f) = \omega v_0\cos\alpha\sin\lambda\,t_f^2$

$\rightarrow d = \omega v_0\cos\alpha\sin\lambda\left(\dfrac{4v_0^2\sin^2\alpha}{g^2}\right)$

$\rightarrow \boxed{d = \dfrac{4v_0^3\omega}{g^2}\cos\alpha\sin^2\alpha\sin\lambda}$