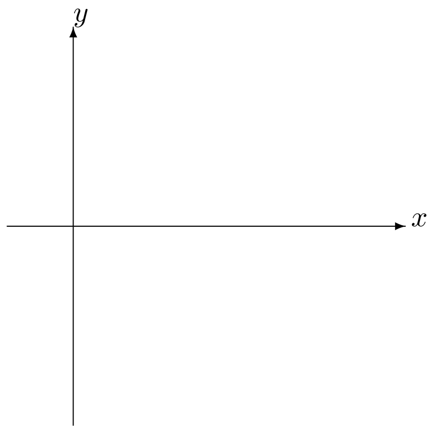
MATH 348 - April 14, 2008
Exam II - 50 Points - 50 minutes

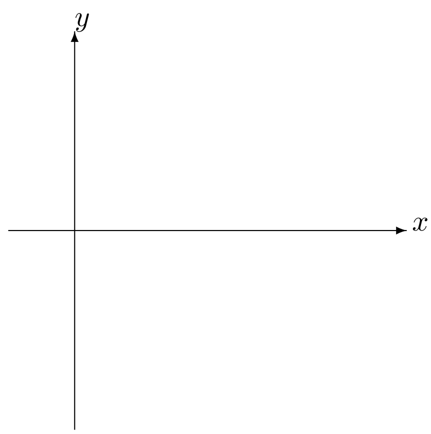
NAME:
SECTION:

In order to receive full credit, SHOW ALL YOUR WORK. Full credit will be given only if all reasoning and work is provided. When applicable, please enclose your final answers in boxes.

1. (10 Points) Conceptual Questions.
   Suppose that we have the graph of the initial condition $u(x, 0) = f(x)$:



   (a) Assume that f is the initial temperature for a homogenous heat problem with boundary conditions, $u_x(0, t) = 0$, $u_x(1, t) = 0$. Describe the physical meaning of these boundary conditions and graph the approximate temperature profile for $t \to \infty$.



   (b) Assume that f is the initial displacement of an elastic string modeled by the homogenous wave equation subject to boundary conditions $u(0, t) = 0$, $u(1, t) = 0$. Describe the physical meaning of these boundary conditions and describe time-dynamics of the points, P_1 and P_2, on this elastic string assuming that the string has no initial velocity.

2. (10 Points)

   (a) Show that $u(x,t) = \dfrac{1}{x - ct}$ is a solution to the one-dimensional wave equation.

   (b) Show that $u(x,y) = \ln(x^2 + y^2)$ is a solution to $u_{xx} + u_{yy} = 0$.

3. (10 Points) Given,

$$\frac{\partial u}{\partial t} + u = \frac{\partial^2 u}{\partial x^2}. \tag{1}$$

   Using separation of variables, $u(x,t) = F(x)G(t)$, and find two ODE's associated with the PDE. Determine the general solution to each of the ODE's, assuming the separation constant $k = 3$.

4. (10 Points) Suppose that the one-dimensional wave equation gives rise to the boundary value problem,

$$F''(x) + kF(x) = 0, \quad k \geq 0, \tag{2}$$

$$F'(0) = 0, \quad F'(1) = 0. \tag{3}$$

(a) Explain the physical interpretation of the boundary conditions (3).

(b) Find all solutions to the the BVP (2)-(3).

5. (10 Points) Suppose that we know that,

$$\begin{aligned} G_n(t) &= B_n e^{-k_n c^2 t}, \quad B_n \in \mathbb{R}, & (4) \\ F_n(x) &= \cos(k_n x), \quad k_n = n\pi, \quad n = 0, 1, 2, \cdots, & (5) \end{aligned}$$

are the temporal and spatial solutions to some heat equation. Assuming that $u(x, 0) = f(x)$:

(a) Write down the general solution to the PDE.

(b) Solve for any unknown constants in terms of $f(x)$.

(c) What is long term behavior of the temperature of this one-dimensional object?