



# Transforming a matrix to reduced row echelon form

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v. 1.23a

## TRANSFORMING MATRIX TO THE REDUCED ROW ECHELON FORM

$$A_1 =$$

Row  
Operation  
1:

$$\begin{array}{cccc} 1 & -3 & 0 & 5 \\ -1 & 1 & 5 & 2 \\ 0 & 1 & 1 & 0 \end{array}$$

add **1** times the 1st row to the 2nd row

$$\begin{array}{cccc} 1 & -3 & 0 & 5 \\ 0 & -2 & 5 & 7 \\ 0 & 1 & 1 & 0 \end{array}$$

Row  
Operation  
2:

$$\begin{array}{cccc} 1 & -3 & 0 & 5 \\ 0 & -2 & 5 & 7 \\ 0 & 1 & 1 & 0 \end{array}$$

multiply the 2nd row by **-1/2**

$$\begin{array}{cccc} 1 & -3 & 0 & 5 \\ 0 & 1 & \frac{-5}{2} & \frac{-7}{2} \\ 0 & 1 & 1 & 0 \end{array}$$

Row  
Operation  
3:

$$\begin{array}{cccc} 1 & -3 & 0 & 5 \\ 0 & 1 & \frac{-5}{2} & \frac{-7}{2} \\ 0 & 1 & 1 & 0 \end{array}$$

add **-1** times the 2nd row to the 3rd row

$$\begin{array}{cccc} 1 & -3 & 0 & 5 \\ 0 & 1 & \frac{-5}{2} & \frac{-7}{2} \\ 0 & 0 & \frac{7}{2} & \frac{7}{2} \end{array}$$

Row  
Operation  
4:

$$\begin{array}{cccc} 1 & -3 & 0 & 5 \\ 0 & 1 & \frac{-5}{2} & \frac{-7}{2} \\ 0 & 0 & \frac{7}{2} & \frac{7}{2} \end{array}$$

multiply the 3rd row by **2/7**

$$\begin{array}{cccc} 1 & -3 & 0 & 5 \\ 0 & 1 & \frac{-5}{2} & \frac{-7}{2} \\ 0 & 0 & 1 & 1 \end{array}$$

**Row  
Operation  
5:**

1	-3	0	5
0	1	$\frac{-5}{2}$	$\frac{-7}{2}$
0	0	1	1

add  $\frac{5}{2}$  times the 3rd row  
to the 2nd row

1	-3	0	5
0	1	0	-1
0	0	1	1

**Row  
Operation  
6:**

1	-3	0	5
0	1	0	-1
0	0	1	1

add **3** times the 2nd row to  
the 1st row

1	0	0	2
0	1	0	-1
0	0	1	1



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$A_2 =$

Row  
Operation  
1:

6	18	-4	20
-1	-3	8	4
5	15	-9	11

multiply the 1st row by  $\frac{1}{6}$

1	3	$\frac{-2}{3}$	$\frac{10}{3}$
-1	-3	8	4
5	15	-9	11

Row  
Operation  
2:

1	3	$\frac{-2}{3}$	$\frac{10}{3}$
		3	3
-1	-3	8	4
5	15	-9	11

add 1 times the 1st row to the 2nd row

1	3	$\frac{-2}{3}$	$\frac{10}{3}$
		3	3
0	0	$\frac{22}{3}$	$\frac{22}{3}$
5	15	-9	11

Row  
Operation  
3:

1	3	$\frac{-2}{3}$	$\frac{10}{3}$
		3	3
0	0	$\frac{22}{3}$	$\frac{22}{3}$
		3	3
5	15	-9	11

add -5 times the 1st row to the 3rd row

1	3	$\frac{-2}{3}$	$\frac{10}{3}$
		3	3
0	0	$\frac{22}{3}$	$\frac{22}{3}$
		3	3
0	0	$\frac{-17}{3}$	$\frac{-17}{3}$
		3	3

Row  
Operation  
4:

1	3	$\frac{-2}{3}$	$\frac{10}{3}$
		3	3

multiply the 2nd row by  $\frac{3}{22}$

1	3	$\frac{-2}{3}$	$\frac{10}{3}$
		3	3

$$\begin{array}{cc|cc} 0 & 0 & 22 & 22 \\ & & 3 & 3 \\ 0 & 0 & -17 & -17 \\ & & 3 & 3 \end{array}$$

$$\begin{array}{cc|cc} 0 & 0 & 1 & 1 \\ & & -17 & -17 \\ 0 & 0 & 3 & 3 \end{array}$$

**Row  
Operation  
5:**

$$\begin{array}{cc|cc} 1 & 3 & -2 & 10 \\ & & 3 & 3 \\ 0 & 0 & 1 & 1 \\ & & -17 & -17 \\ 0 & 0 & 3 & 3 \end{array}$$

add  $\mathbf{17/3}$  times the 2nd row to the 3rd row

$$\begin{array}{cc|cc} & & -2 & 10 \\ 1 & 3 & 3 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}$$

**Row  
Operation  
6:**

$$\begin{array}{cc|cc} 1 & 3 & -2 & 10 \\ & & 3 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}$$

add  $\mathbf{2/3}$  times the 2nd row to the 1st row

$$\begin{array}{cc|cc} 1 & 3 & 0 & 4 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}$$



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## TRANSFORMING MATRIX TO THE REDUCED ROW ECHELON FORM

$A_3 =$

<b>Row Operation 1:</b>	$\begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 1 & 0 & 3 & 0 \end{bmatrix}$	add <b>-1</b> times the 1st row to the 3rd row	$\begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & -2 & 2 & -4 \end{bmatrix}$
<b>Row Operation 2:</b>	$\begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & -2 & 2 & -4 \end{bmatrix}$	add <b>2</b> times the 2nd row to the 3rd row	$\begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & -2 \end{bmatrix}$
<b>Row Operation 3:</b>	$\begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & -2 \end{bmatrix}$	multiply the 3rd row by <b>-1/2</b>	$\begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
<b>Row Operation 4:</b>	$\begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	add <b>-1</b> times the 3rd row to the 2nd row	$\begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
<b>Row Operation 5:</b>	$\begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	add <b>-4</b> times the 3rd row to the 1st row	$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
<b>Row Operation 6:</b>	$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix}$	add <b>-2</b> times the 2nd row to the 1st row	$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix}$

$$\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$$



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## TRANSFORMING MATRIX TO THE REDUCED ROW ECHELON FORM

$A_4 =$   
**Row Operation 1:**

1	2	3	10
2	4	6	20
3	6	9	30

 add **-2** times the 1st row to the 2nd row
 

1	2	3	10
0	0	0	0
3	6	9	30

**Row Operation 2:**

1	2	3	10
0	0	0	0
3	6	9	30

 add **-3** times the 1st row to the 3rd row
 

1	2	3	10
0	0	0	0
0	0	0	0



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## TRANSFORMING MATRIX TO THE REDUCED ROW ECHELON FORM

$A_5 =$

Row  
Operation  
1:

5	3	22
-4	7	20
9	-2	15

multiply the 1st row by  $1/5$

1	$\frac{3}{5}$	$\frac{22}{5}$
-4	7	20
9	-2	15

Row  
Operation  
2:

1	$\frac{3}{5}$	$\frac{22}{5}$
-4	7	20
9	-2	15

add 4 times the 1st row to the 2nd row

1	$\frac{3}{5}$	$\frac{22}{5}$
0	5	5
0	47	188
9	-2	15

Row  
Operation  
3:

1	$\frac{3}{5}$	$\frac{22}{5}$
0	5	5
0	47	188
9	-2	15

add -9 times the 1st row to the 3rd row

1	$\frac{3}{5}$	$\frac{22}{5}$
0	5	5
0	47	188
0	5	5
0	-37	-123
0	5	5

Row  
Operation  
4:

1	$\frac{3}{5}$	$\frac{22}{5}$
0	5	5

multiply the 2nd row by  $5/47$

1	$\frac{3}{5}$	$\frac{22}{5}$
0	5	5



$$\begin{array}{r|rr} 0 & 47 & 188 \\ & \hline & 5 & 5 \\ 0 & -37 & -123 \\ & \hline & 5 & 5 \end{array}$$

$$\begin{array}{r|rr} 0 & 1 & 4 \\ & \hline & -37 & -123 \\ 0 & \hline & 5 & 5 \end{array}$$

**Row  
Operation  
5:**

$$\begin{array}{r|rr} 1 & 3 & 22 \\ & \hline & 5 & 5 \\ 0 & 1 & 4 \\ & \hline & -37 & -123 \\ 0 & \hline & 5 & 5 \end{array}$$

add  $\frac{37}{5}$  times the 2nd row to the 3rd row

$$\begin{array}{r|rr} & 3 & 22 \\ 1 & \hline & 5 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 5 \end{array}$$

**Row  
Operation  
6:**

$$\begin{array}{r|rr} 1 & 3 & 22 \\ & \hline & 5 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 5 \end{array}$$

multiply the 3rd row by  $\frac{1}{5}$

$$\begin{array}{r|rr} 1 & 3 & 22 \\ & \hline & 5 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{array}$$

**Row  
Operation  
7:**

$$\begin{array}{r|rr} 1 & 3 & 22 \\ & \hline & 5 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{array}$$

add  $-4$  times the 3rd row to the 2nd row

$$\begin{array}{r|rr} 1 & 3 & 22 \\ & \hline & 5 & 5 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}$$

**Row  
Operation  
8:**

$$\begin{array}{r|rr} 1 & 3 & 22 \\ & \hline & 5 & 5 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}$$

add  $-\frac{22}{5}$  times the 3rd row to the 1st row

$$\begin{array}{r|rr} & 3 & \\ 1 & \hline & 5 & \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}$$

**Row  
Operation  
9:**

$$\begin{array}{r|rr} 1 & 3 & \\ & \hline & 0 & \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}$$

add  $-\frac{3}{5}$  times the 2nd row to the 1st row

$$\begin{array}{r|rr} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}$$

$$\begin{bmatrix} & & 5 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} 1 & -5 & 1 \\ -1 & 7 & 1 \\ -3 & 8 & h \end{bmatrix} \quad \begin{array}{l} R_2 = R_2 + R_1 \\ R_3 = R_3 + 3R_1 \end{array} \sim \begin{bmatrix} 1 & -5 & 1 \\ 0 & 2 & 2 \\ 0 & -7 & h+3 \end{bmatrix} \sim$$

$$R_3 = \frac{R_3}{2} \sim \begin{bmatrix} 1 & -5 & 1 \\ 0 & 1 & 1 \\ 0 & -7 & h+3 \end{bmatrix} \quad R_3 = R_3 + 7R_2 \sim \begin{bmatrix} 1 & -5 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & h+10 \end{bmatrix}$$

• If  $h \neq -10$  then

$$R_3 = \frac{R_3}{h+10} \sim \begin{bmatrix} 1 & -5 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{array}{l} R_2 = R_2 - R_3 \\ R_1 = R_1 - R_3 \end{array} \sim \begin{bmatrix} 1 & -5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim$$

$$R_1 = R_1 + 5R_2 \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

• If  $h = -10$  then

$$R_1 = R_1 + 5R_2 \sim \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A_7 = \begin{bmatrix} 1 & -3 & 5 \\ -3 & 9 & 7 \\ 2 & -6 & h \end{bmatrix} \begin{array}{l} R_2 = R_2 + 3R_1 \\ \sim \\ R_3 = R_3 - 2R_1 \end{array} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 0 & 8 \\ 0 & 0 & h-10 \end{bmatrix} \sim$$

$$\begin{array}{l} R_3 = R_2 \\ \sim \\ R_2 = R_3 \end{array} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 0 & h-10 \\ 0 & 0 & 8 \end{bmatrix} \begin{array}{l} R_1 = R_1 - \frac{5}{8}R_3 \\ \sim \\ R_2 = R_2 - \frac{(h-10)R_3}{8} \end{array} \begin{bmatrix} 1 & -3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 8 \end{bmatrix} \sim$$

$$\begin{array}{l} R_2 = R_3/8 \\ \sim \\ R_3 = R_2 \end{array} \begin{bmatrix} 1 & -3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$



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## TRANSFORMING MATRIX TO THE REDUCED ROW ECHELON FORM

$A_8 =$

<b>Row Operation 1:</b>	$\begin{bmatrix} 1 & 2 & 4 & 3 \\ 0 & 1 & 2 & 1 \\ -1 & 3 & 6 & 2 \end{bmatrix}$	add <b>1</b> times the 1st row to the 3rd row	$\begin{bmatrix} 1 & 2 & 4 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 5 & 10 & 5 \end{bmatrix}$
<b>Row Operation 2:</b>	$\begin{bmatrix} 1 & 2 & 4 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 5 & 10 & 5 \end{bmatrix}$	add <b>-5</b> times the 2nd row to the 3rd row	$\begin{bmatrix} 1 & 2 & 4 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
<b>Row Operation 3:</b>	$\begin{bmatrix} 1 & 2 & 4 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$	add <b>-2</b> times the 2nd row to the 1st row	$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$



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## TRANSFORMING MATRIX TO THE REDUCED ROW ECHELON FORM

Row  
Operation  
1:

$$A_9 = \begin{bmatrix} -8 & -2 & -9 & 2 \\ 6 & 4 & 8 & 1 \\ 4 & 0 & 4 & -2 \end{bmatrix}$$

multiply the 1st row by  $-\frac{1}{8}$

$$\begin{bmatrix} 1 & \frac{9}{4} & -\frac{1}{4} & -\frac{1}{4} \\ 6 & 4 & 8 & 1 \\ 4 & 0 & 4 & -2 \end{bmatrix}$$

Row  
Operation  
2:

$$\begin{bmatrix} 1 & \frac{9}{4} & -\frac{1}{4} & -\frac{1}{4} \\ 6 & 4 & 8 & 1 \\ 4 & 0 & 4 & -2 \end{bmatrix}$$

add  $-6$  times the 1st row to the 2nd row

$$\begin{bmatrix} 1 & \frac{9}{4} & -\frac{1}{4} & -\frac{1}{4} \\ 0 & 5 & 5 & 5 \\ 4 & 0 & 4 & -2 \end{bmatrix}$$

Row  
Operation  
3:

$$\begin{bmatrix} 1 & \frac{9}{4} & -\frac{1}{4} & -\frac{1}{4} \\ 0 & 5 & 5 & 5 \\ 4 & 0 & 4 & -2 \end{bmatrix}$$

add  $-4$  times the 1st row to the 3rd row

$$\begin{bmatrix} 1 & \frac{9}{4} & -\frac{1}{4} & -\frac{1}{4} \\ 0 & 5 & 5 & 5 \\ 0 & -1 & 2 & -\frac{1}{2} \end{bmatrix}$$

Row  
Operation  
4:

$$\begin{bmatrix} 1 & \frac{9}{4} & -\frac{1}{4} & -\frac{1}{4} \\ 0 & \frac{2}{5} & \frac{2}{5} & \frac{2}{5} \\ 0 & -1 & 2 & -\frac{1}{2} \end{bmatrix}$$

multiply the 2nd row by  $\frac{5}{2}$

$$\begin{bmatrix} 1 & \frac{9}{4} & -\frac{1}{4} & -\frac{1}{4} \\ 0 & 2 & 2 & 2 \\ 0 & -1 & 2 & -\frac{1}{2} \end{bmatrix}$$





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## TRANSFORMING MATRIX TO THE REDUCED ROW ECHELON FORM

$A_{10} =$

Row  
Operation  
1:

$$\begin{array}{ccc|c} 3 & 6 & 7 & \\ \hline 0 & 2 & 1 & \\ 2 & 3 & 4 & \end{array}$$

multiply the 1st row by  $\frac{1}{3}$

$$\begin{array}{ccc|c} 1 & 2 & \frac{7}{3} & \\ \hline 0 & 2 & 1 & \\ 2 & 3 & 4 & \end{array}$$

Row  
Operation  
2:

$$\begin{array}{ccc|c} 1 & 2 & \frac{7}{3} & \\ \hline 0 & 2 & 1 & \\ 2 & 3 & 4 & \end{array}$$

add  $-2$  times the 1st row to the 3rd row

$$\begin{array}{ccc|c} 1 & 2 & \frac{7}{3} & \\ \hline 0 & 2 & 1 & \\ 0 & -1 & \frac{-2}{3} & \end{array}$$

Row  
Operation  
3:

$$\begin{array}{ccc|c} 1 & 2 & \frac{7}{3} & \\ \hline 0 & 2 & 1 & \\ 0 & -1 & \frac{-2}{3} & \end{array}$$

multiply the 2nd row by  $\frac{1}{2}$

$$\begin{array}{ccc|c} 1 & 2 & \frac{7}{3} & \\ \hline 0 & 1 & \frac{1}{2} & \\ 0 & -1 & \frac{-2}{3} & \end{array}$$

Row  
Operation  
4:

$$\begin{array}{ccc|c} 1 & 2 & \frac{7}{3} & \\ \hline 0 & 1 & \frac{1}{2} & \\ 0 & -1 & \frac{-2}{3} & \end{array}$$

add  $1$  times the 2nd row to the 3rd row

$$\begin{array}{ccc|c} 1 & 2 & \frac{7}{3} & \\ \hline 0 & 1 & \frac{1}{2} & \\ 0 & 0 & \frac{-1}{6} & \end{array}$$



$$\begin{array}{ccc} & & 1 \\ 0 & 1 & \hline & & 2 \\ 0 & -1 & \hline & & 3 \end{array}$$

$$\begin{array}{ccc} & & 1 \\ 0 & 1 & \hline & & 2 \\ 0 & 0 & \hline & & -1 \\ & & 6 \end{array}$$

**Row  
Operation  
5:**

$$\begin{array}{ccc} & & 7 \\ 1 & 2 & \hline & & 3 \\ & & 1 \\ 0 & 1 & \hline & & 2 \\ & & -1 \\ 0 & 0 & \hline & & 6 \end{array}$$

multiply the 3rd row by **-6**

$$\begin{array}{ccc} & & 7 \\ 1 & 2 & \hline & & 3 \\ & & 1 \\ 0 & 1 & \hline & & 2 \\ 0 & 0 & 1 \end{array}$$

**Row  
Operation  
6:**

$$\begin{array}{ccc} & & 7 \\ 1 & 2 & \hline & & 3 \\ & & 1 \\ 0 & 1 & \hline & & 2 \\ & & 1 \\ 0 & 0 & 1 \end{array}$$

add **-1/2** times the 3rd row to the 2nd row

$$\begin{array}{ccc} & & 7 \\ 1 & 2 & \hline & & 3 \\ & & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}$$

**Row  
Operation  
7:**

$$\begin{array}{ccc} & & 7 \\ 1 & 2 & \hline & & 3 \\ & & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}$$

add **-7/3** times the 3rd row to the 1st row

$$\begin{array}{ccc} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}$$

**Row  
Operation  
8:**

$$\begin{array}{ccc} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}$$

add **-2** times the 2nd row to the 1st row

$$\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}$$



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## TRANSFORMING MATRIX TO THE REDUCED ROW ECHELON FORM

$$A_{11} =$$

Row Operation 1:

2	-3	6	2	5
-2	3	-3	-3	-4
4	-6	9	5	9
-2	3	3	-4	1

multiply the 1st row by  $\frac{1}{2}$

1	$\frac{-3}{2}$	3	1	$\frac{5}{2}$
-2	3	-3	-3	-4
4	-6	9	5	9
-2	3	3	-4	1

Row Operation 2:

1	$\frac{-3}{2}$	3	1	$\frac{5}{2}$
-2	3	-3	-3	-4
4	-6	9	5	9
-2	3	3	-4	1

add 2 times the 1st row to the 2nd row

1	$\frac{-3}{2}$	3	1	$\frac{5}{2}$
0	0	3	-1	1
4	-6	9	5	9
-2	3	3	-4	1

Row Operation 3:

1	$\frac{-3}{2}$	3	1	$\frac{5}{2}$
0	0	3	-1	1
4	-6	9	5	9
-2	3	3	-4	1

add -4 times the 1st row to the 3rd row

1	$\frac{-3}{2}$	3	1	$\frac{5}{2}$
0	0	3	-1	1
0	0	-3	1	-1
-2	3	3	-4	1

Row Operation 4:

1	$\frac{-3}{2}$	3	1	$\frac{5}{2}$
0	0	3	-1	1

add 2 times the 1st row to the 4th row

1	$\frac{-3}{2}$	3	1	$\frac{5}{2}$
0	0	3	-1	1

$$\begin{array}{ccccc} 0 & 0 & -3 & 1 & -1 \\ -2 & 3 & 3 & -4 & 1 \end{array}$$

$$\begin{array}{ccccc} 0 & 0 & -3 & 1 & -1 \\ 0 & 0 & 9 & -2 & 6 \end{array}$$

**Row  
Operation  
5:**

$$\begin{array}{ccccc} & -3 & & & 5 \\ 1 & \frac{\quad}{2} & 3 & 1 & \frac{\quad}{2} \\ 0 & 0 & 3 & -1 & 1 \\ 0 & 0 & -3 & 1 & -1 \\ 0 & 0 & 9 & -2 & 6 \end{array}$$

multiply the 2nd row by  $\frac{1}{3}$

$$\begin{array}{ccccc} & -3 & & & 5 \\ 1 & \frac{\quad}{2} & 3 & 1 & \frac{\quad}{2} \\ & & & -1 & 1 \\ 0 & 0 & 1 & \frac{\quad}{3} & \frac{\quad}{3} \\ & & & 3 & 3 \\ 0 & 0 & -3 & 1 & -1 \\ 0 & 0 & 9 & -2 & 6 \end{array}$$

**Row  
Operation  
6:**

$$\begin{array}{ccccc} & -3 & & & 5 \\ 1 & \frac{\quad}{2} & 3 & 1 & \frac{\quad}{2} \\ & & & -1 & 1 \\ 0 & 0 & 1 & \frac{\quad}{3} & \frac{\quad}{3} \\ & & & 3 & 3 \\ 0 & 0 & -3 & 1 & -1 \\ 0 & 0 & 9 & -2 & 6 \end{array}$$

add 3 times the 2nd row to the 3rd row

$$\begin{array}{ccccc} & -3 & & & 5 \\ 1 & \frac{\quad}{2} & 3 & 1 & \frac{\quad}{2} \\ & & & -1 & 1 \\ 0 & 0 & 1 & \frac{\quad}{3} & \frac{\quad}{3} \\ & & & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 9 & -2 & 6 \end{array}$$

**Row  
Operation  
7:**

$$\begin{array}{ccccc} & -3 & & & 5 \\ 1 & \frac{\quad}{2} & 3 & 1 & \frac{\quad}{2} \\ & & & -1 & 1 \\ 0 & 0 & 1 & \frac{\quad}{3} & \frac{\quad}{3} \\ & & & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 9 & -2 & 6 \end{array}$$

add -9 times the 2nd row to the 4th row

$$\begin{array}{ccccc} & -3 & & & 5 \\ 1 & \frac{\quad}{2} & 3 & 1 & \frac{\quad}{2} \\ & & & -1 & 1 \\ 0 & 0 & 1 & \frac{\quad}{3} & \frac{\quad}{3} \\ & & & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 \end{array}$$

**Row  
Operation  
8:**

$$\begin{array}{ccccc} & -3 & & & 5 \\ 1 & \frac{\quad}{2} & 3 & 1 & \frac{\quad}{2} \\ & & & -1 & 1 \\ 0 & 0 & 1 & \frac{\quad}{\quad} & \frac{\quad}{\quad} \end{array}$$

interchange the 3rd row and the 4th row

$$\begin{array}{ccccc} & -3 & & & 5 \\ 1 & \frac{\quad}{2} & 3 & 1 & \frac{\quad}{2} \\ & & & -1 & 1 \\ 0 & 0 & 1 & \frac{\quad}{\quad} & \frac{\quad}{\quad} \end{array}$$

$$\begin{array}{ccccc} & & & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 \end{array}$$

$$\begin{array}{ccccc} & & & 3 & 3 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

**Row  
Operation  
9:**

$$\begin{array}{ccccc} 1 & \frac{-3}{2} & 3 & 1 & \frac{5}{2} \\ 0 & 0 & 1 & \frac{-1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

add  $\frac{1}{3}$  times the 3rd row to the 2nd row

$$\begin{array}{ccccc} 1 & \frac{-3}{2} & 3 & 1 & \frac{5}{2} \\ 0 & 0 & 1 & 0 & \frac{4}{3} \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

**Row  
Operation  
10:**

$$\begin{array}{ccccc} 1 & \frac{-3}{2} & 3 & 1 & \frac{5}{2} \\ 0 & 0 & 1 & 0 & \frac{4}{3} \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

add  $-1$  times the 3rd row to the 1st row

$$\begin{array}{ccccc} 1 & \frac{-3}{2} & 3 & 0 & \frac{-1}{2} \\ 0 & 0 & 1 & 0 & \frac{4}{3} \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

**Row  
Operation  
11:**

$$\begin{array}{ccccc} 1 & \frac{-3}{2} & 3 & 0 & \frac{-1}{2} \\ 0 & 0 & 1 & 0 & \frac{4}{3} \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

add  $-3$  times the 2nd row to the 1st row

$$\begin{array}{ccccc} 1 & \frac{-3}{2} & 0 & 0 & \frac{-9}{2} \\ 0 & 0 & 1 & 0 & \frac{4}{3} \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$