<u>Note:</u> Linear Independence

If $b^2 - 4km \neq 0$ then $\lambda_1 \neq \lambda_2$

$$y(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$$

Since

$$W = \det\begin{pmatrix} e^{\lambda_1 t} & e^{\lambda_2 t} \\ \lambda_1 e^{\lambda_1 t} & \lambda_2 e^{\lambda_2 t} \end{pmatrix} = e^{\lambda_1 t} e^{\lambda_2 t} (\lambda_2 - \lambda_1) \neq 0$$

for $b^2 - 4km = 0$

$$y_2(t) = te^{\lambda t} \Rightarrow my_2'' + by_2' + ky_2 =$$

$$= m(2\lambda e^{\lambda t} + \lambda^2 te^{\lambda t}) + b(\lambda te^{\lambda t} + e^{\lambda t}) +$$

$$+ kte^{\lambda t} = \underbrace{(m\lambda^2 + b\lambda + k)}_{=0 \text{ by Quad. Eqn}} te^{\lambda t} + (2\lambda m + b)e^{\lambda t} =$$

$$= \left(2\left(-\frac{b}{2m}\right)\cdot m + b\right)e^{\lambda t} = (-b+b)e^{\lambda t} = 0$$

$\Rightarrow$ y_2 is a sol<u>n</u>

Also

$$W = \det\begin{pmatrix} e^{\lambda t} & te^{\lambda t} \\ \lambda e^{\lambda t} & e^{\lambda t} + \lambda te^{\lambda t} \end{pmatrix} =$$

$$= \lambda te^{2\lambda t} - \lambda te^{2\lambda t} + e^{2\lambda t} = e^{2\lambda t} \neq 0 \text{ for any } t.$$

$\Rightarrow$

$$y(t) = C_1 e^{\lambda t} + C_2 te^{\lambda t}$$

is the gen. sol<u>n</u>