Homework 4 due 21 Sept. 2007 in class
PH462 EM Waves and Optical Physics posted: 14 Sept 2007

1) Problem 3 from the last homework set (optical activity).

2) As we discussed in class, focused laser beams can trap particles by redirecting rays of light. You will show here that a perfectly transmitting, refractive sphere will experience a transverse force toward higher intensity. For this problem, we consider a perfectly transmitting sphere with an index of refraction *n* in air, and we'll do a simple calculation using ray optics.
   a. Trace rays through the sphere for input rays that are parallel to the z-axis. Calculate an expression for the angle of incidence on the sphere in the x-z plane vs ray height x. Note that this angle will change sign along with x. Use Snell's Law to calculate the refracted angle, and geometry to figure out the incidence angle on the second surface. You will also need an expression for the net angle of deflection between the incident ray and the ray transmitted through the sphere.
   b. The pressure vector of a single beam at the interface is $\mathbf{P} = \mathbf{S}\cos[\theta]/c$, where S is the local Poynting vector. Calculate the difference between the pressures of the outgoing waves and the incoming waves as a function of the input ray height x.
   c. Let $\mathbf{S}_{in}(x) = \hat{\mathbf{z}} I_o (1 + \alpha x)$, with $\alpha > 0$. Integrate the pressure difference over x to get the net x and z components of the force. (In this simpler version, you are really doing a calculation for a cylinder). Show that the net transverse force on the sphere is in the positive x direction.
   d. Qualitatively, describe the effect of reflection at the first surface. Briefly, discuss what you would have to do to make the calculation more realistic.

3) A filter to reduce the intensity of a beam can be made by evaporating a thin layer of metal, such as silver, onto a glass substrate. In the lab these are typically to control the light level on a detector or a camera or as beamsplitters. The real and imaginary parts of the refractive index for silver for green light (λ = 500nm), are $n_R = 0.15$ and $n_I = 4.0$.
   a. Calculate σ and ε from these values of the index. Note that at this high frequency the result differs from the DC values.
   b. Calculate the skin depth δ for green light incident on silver. Use the convention that the skin depth is where the *field* falls off by 1/e from the surface.
   c. To calculate the transmission through a thin layer ($L \approx \delta$), one must in principle account for the reflection at the second metal-glass interface. But that reflected wave is attenuated on the way back, so we can to a good approximation ignore that second reflection. Assuming then that the transmitted intensity is $I_t = I_{inc} e^{-\alpha L}$. The attenuation coefficient is $\alpha = 2/\delta$ (the factor of 2 is there because we are concerned with the intensity). The "optical density" (OD) of a filter is defined as $-\log_{10}\left(I_t / I_{inc} \right)$. Calculate the thickness of a silver film required to make filters of 0.1, 0.5, 1.0, 2.0 and 3.0 OD (or plot the thickness vs. OD over this range).

4) HM problem 6-3
5) HM problem 6-4