PHGN 462 Homework 3

1) Pollack and Stump 11.13. Standing waves in cavities show up in a lot of practical places; not the least of which is a laser cavity.

For part a, comment briefly on the fact that our old familiar $E = cB$ doesn't seem to apply here.

Note that part b rather cavalierly asks you to sketch the fields in this box. It's asking you to sketch fairly complicated E's and B's that occupy a three-dimensional space. That's not simple. In fact, I'd like you to give particular thought to how best to visually represent those fields in this situation. There is no one accepted standard.

2) There's a general method for describing the polarization of electromagnetic waves using what are called Jones vectors. We'll restrict ourselves to describing the electric field in a wave, since once you know E and the direction in which the wave is traveling, you can easily find the orientation and amplitude of B (at least, I hope you can).

Let's say we have an E-field propagating along the z-axis with the form

$$\vec{E} = E_x e^{i(kz-wt-\varphi_x)}\hat{\imath} + E_y e^{i(kz-wt-\varphi_y)}\hat{\jmath}$$

Such a form is totally general, and allows for the possibility that the $\hat{\imath}$ and $\hat{\jmath}$ components of the field are of different amplitudes and also of different phases with respect to one another. We can factor out the part common to both components and write $\vec{E}$ in vector form as follows:

$$\vec{E} = e^{i(kz-wt)}\begin{pmatrix} E_x e^{i\varphi_x} \\ E_y e^{i\varphi_y} \end{pmatrix}$$

The term in parentheses has all the information that's unique to a particular field, showing both the component amplitudes and phases. This term is the Jones vector.

The Jones vectors for horizontally and vertically polarized light with unit amplitude are $\overrightarrow{E_x} = \binom{1}{0}$ and $\overrightarrow{E_y} = \binom{0}{1}$, respectively. This pair defines a simple basis that can be used to express the electric field for any wave.

a) Consider the E-field whose real part is $\vec{E} = 7\cos(kz - wt)\,\hat{\imath} - 5\sin(kz - wt)\,\hat{\jmath}$ (with implied units attached to the 7 and 5). Figure out how to decompose this field in terms of $\overrightarrow{E_x}$ and $\overrightarrow{E_y}$. By that I mean find the A and B such that $\vec{E} = A\overrightarrow{E_x} + B\overrightarrow{E_y}$. You may find it helpful to start by expressing $\vec{E}$ in terms of a Jones vector. And don't forget that you can use imaginary coefficients if you need to.

b) An alternative basis for describing polarization is referred to as circular polarization. The basis vectors (in Jones notation) are $\overrightarrow{E_L} = \frac{1}{\sqrt{2}}\binom{1}{i}$ and $\overrightarrow{E_R} = \frac{1}{\sqrt{2}}\binom{1}{-i}$, describing left-circular and right-circular polarization respectively.

Explain, using an appropriate combination of words, equations, and diagrams, why this basis is referred to as *circular* polarization. It's no great challenge to find the answer on the interwebs, so make sure your explanation is strong and is uniquely your own. Also mention why those $\frac{1}{\sqrt{2}}$ factors are there.

c) Express the E-field from part (a) in terms of $\overrightarrow{E_L}$ and $\overrightarrow{E_R}$.

Two things to take away from this problem: 1) How to express polarization in general and 2) That circular polarization, while it sometimes sounds like an odd thing, is just another basis to work in, one that happens to come up a lot in optics labs.

3) Pollack and Stump 11.34. This problem gives us a look at the actual mechanical effects of fields on particles, and shows us how to treat the general problem numerically (real problems almost always have numerical solutions). Some clarifications/hints follow:

For c & d you have to be super careful with the difference between total and partial derivatives, and attend to the fact that f is a function of $(x - ct)$. It's tricky. Fortunately, it's a "show that" problem, so if you get stuck on c & d you can still do the rest. I found it useful to define $\eta = x - ct$ so as to write $f(x - ct)$ as $f(\eta)$. That helped me keep track of all the multivariate chain rule stuff. Though I still had to write things out in almost embarrassing detail to get all the terms to show up right.

For e I'm not really sure what pictures you could draw that aren't redundant with f & g, so if you're not sure what to draw, don't sweat it.

For f, I think it's implied that $\xi = x - ct$, even though they kind of forgot to actually say that. They also leave "relatively small" depressingly vague. Let's say "relatively small" is 0.1.

For f also note that if they say, for example, that d is a "unit" of length, that means d has unit value. Which is to say, it's 1. And if d is unit and d/c is unit, so must be c. Really, we're just trying to get rid of all the constants so we can zoom in on the qualitative behavior of the system.

4) Pollack and Stump 11.28. And in part (a) prove that $\frac{dP}{d\Omega} = r^2 \overrightarrow{S_{avg}} \cdot \hat{r}$ (it's a fairly short proof).

Also add part c:

c) So this "spherical wave" obviously doesn't have spherical symmetry. Something about the physical situation must break the symmetry if we don't get a spherically symmetric solution. What is it?