

plate set in d

1. Using separation of variables, derive an expression for the voltage between an infinite parallel plate capacitor with voltage V_0 on the upper plate and grounded on the lower plate. For credit you need to justify all the steps starting from Laplace's equation.

$$\nabla^2 V = 0 \quad V = \bar{X}(x) \bar{Y}(y) \bar{Z}(z)$$

$$\cancel{\frac{\partial^2 V}{\partial x^2}} + \cancel{\frac{\partial^2 V}{\partial y^2}} + \cancel{\frac{\partial^2 V}{\partial z^2}} = 0 \quad \therefore \text{by } V \Rightarrow$$

$$\underbrace{\frac{1}{C_1} \frac{\partial^2 \bar{X}}{\partial x^2}}_{C_1} + \underbrace{\frac{1}{C_2} \frac{\partial^2 \bar{Y}}{\partial y^2}}_{C_2} + \underbrace{\frac{1}{C_3} \frac{\partial^2 \bar{Z}}{\partial z^2}}_{C_3} = 0$$

$C_1 + C_2 + C_3 = 0$ since
each term could vary but others
not vary and so could never add to
zero,

\therefore ODE's $\frac{d^2 \bar{X}}{dx^2} = C_1 \bar{X}$; $\frac{d^2 \bar{Y}}{dy^2} = C_2 \bar{Y}$; $\frac{d^2 \bar{Z}}{dz^2} = C_3 \bar{Z}$

∞ plate along $x \Rightarrow$ voltage $\bar{X} = \text{const} \Rightarrow C_1 = 0$
for all points along x must be same $\Rightarrow C_2 = 0$ Now $C_1 + C_2 + C_3 = 0 \Rightarrow C_3 = 0$

Same for $y \Rightarrow C_2 = 0$ $\bar{Y} = A y + B$

left with $\frac{d^2 \bar{Z}}{dz^2} = 0$

$$\bar{Z}(z=0) = 0 \Rightarrow B = 0$$

$$\bar{Z}(z=d) = A d = V_0 \Rightarrow A = \frac{V_0}{d}$$

Boundary conditions

$$V = \bar{X} \bar{Y} \bar{Z} = \frac{V_0}{d} x$$