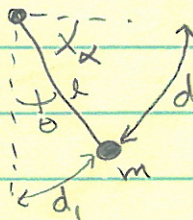


PH 350 Fall 2011 Practice Test 2 solus

1) a) There is only 1 degree of freedom, described by the angle as the pendulum swings back and forth.

b) $\theta, x, d, z = l\theta$ or $dz = lx$ would all give you relatively simple Lagrangians of only 1 coordinate. Not and! Only 1 coordinate is used.



c) Use θ : $x = l \sin \theta$; $y = -l \cos \theta + v_{oy}t - \frac{1}{2}at^2$
 $\dot{x} = l \cos \theta \dot{\theta}$; $\dot{y} = +l \sin \theta \dot{\theta} + v_{oy} - at$

$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{y}^2 = \frac{1}{2}m l^2 \dot{\theta}^2 + m(v_{oy} - at)l \sin \theta \dot{\theta} + \frac{1}{2}m(v_{oy} - at)^2$$

$$U = mgy = -mgl \cos \theta + mgv_{oy}t - \frac{1}{2}mga^2t^2 \quad \{I \text{ used } \sin^2 \theta + \cos^2 \theta = 1\}$$

$$\rightarrow L = \frac{1}{2}m l^2 \dot{\theta}^2 + m(v_{oy} - at)l \sin \theta \dot{\theta} + \frac{1}{2}m(v_{oy} - at)^2 + mgl \cos \theta - mgv_{oy}t + \frac{1}{2}mga^2t^2$$

$$d) H = \frac{\partial L}{\partial \dot{\theta}} \dot{\theta} - L = (m l^2 \dot{\theta} + 2(v_{oy} - at)l \sin \theta) \dot{\theta} - L$$

$$\rightarrow H = \frac{1}{2}m l^2 \dot{\theta}^2 - mgl \cos \theta - (v_{oy} - at)^2 + mgv_{oy}t - \frac{1}{2}mga^2t^2$$

To get $H(\theta, p_\theta)$ {I could ask that on the test}, $p_\theta = \frac{\partial L}{\partial \dot{\theta}} = m l^2 \dot{\theta} + 2(v_{oy} - at)l \sin \theta$

$$\rightarrow \dot{\theta} = \frac{p_\theta}{m l^2} - 2(v_{oy} - at)l \sin \theta$$

$$\rightarrow H(\theta, p_\theta) = \frac{1}{2}m l^2 \left[\frac{p_\theta}{m l^2} - 2(v_{oy} - at)l \sin \theta \right]^2 - mgl \cos \theta - (v_{oy} - at)^2 + mgv_{oy}t - \frac{1}{2}mga^2t^2$$

e) $\frac{dH}{dt} = \frac{\partial H}{\partial t} \neq 0 \rightarrow$ No it's not conserved.

f) No, because $y(\theta; t)$ depends on t .

g) I don't like (g) as much as I used to.

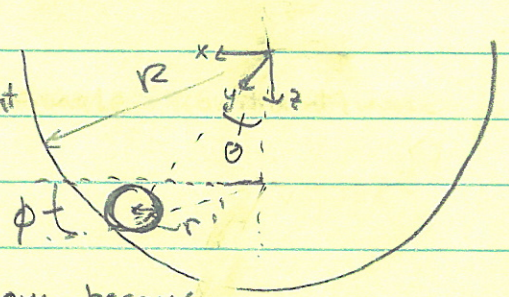
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2) a) b) : 3 rotational and 3 translational.

b) An object constrained to any 2-D surface is constrained in one translational direction (it can't come off the surface)
 → 1 translational constraint.

This could be written as the single constraint equation

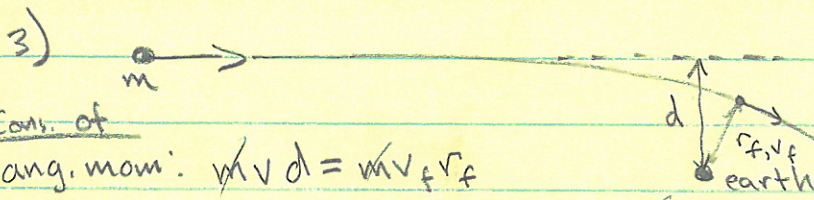
$$x_{cm}^2 + y_{cm}^2 + z_{cm}^2 = (R-r)^2$$



Rotationally you lose all degrees of freedom because if it's not slipping then $\omega_{\perp \text{ plane of motion}} = \frac{v}{r}$ and the two components of ω in the plane of motion $= \phi$. → 3 constraints.
3 rotational constraints

4 constraints → 2 deg. of freedom

c) See the above picture: I would pick the standard θ, ϕ from spherical coordinates. Anything proportional to those will be identical. If you chose something different, be sure you motivate why you chose it. For instance my coords. give simple transformations:
 $x = (R-r) \sin \theta \cos \phi$; $y = (R-r) \sin \theta \sin \phi$
 $z = (R-r) \cos \theta$



Cons. of ang. mom: $m v d = m v_f r_f$

Energy : $\frac{1}{2} m v^2 = -\frac{G M m}{r_f} + \frac{1}{2} m v_f^2$
 $u = 0 @ \infty$

$$\rightarrow r_f = \frac{v d}{v_f} \rightarrow \frac{1}{2} m v^2 = -\frac{G M m}{\frac{v^2 d^2}{v_f^2}} + \frac{1}{2} m v_f^2 = \left(-\frac{G M M}{v^2 d^2} + \frac{1}{2} m \right) v_f^2$$

(see next page)

Note! I find out this is wrong later.

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3 cont'd) $\rightarrow \frac{1}{2}mv^2 = \left(\frac{-GMm}{v^2 d^2} + \frac{1}{2}m \right) v_f^2$

~~$\rightarrow v_f^2 = \frac{v^2}{\left(1 - \frac{2GM}{v^2 d^2}\right)}$~~

or ~~$\rightarrow v_f = \frac{v}{\sqrt{1 - \frac{2GM}{v^2 d^2}}}$~~ $\rightarrow r_f = \frac{vd}{v_f} = d \sqrt{1 - \frac{2GM}{v^2 d^2}}$

Units: $\frac{GM}{d^2} = [m/s^2] \rightarrow \frac{GM}{v^2 d^2} = [1/m]$ doesn't work. Hmm.

Dops $u \neq \frac{-GMm}{r^2} \rightarrow u = \frac{-GMm}{r}$. Starting over

$\rightarrow \frac{1}{2}mv^2 = -\frac{GMm}{r_f} + \frac{1}{2}mv_f^2$; $r_f = \frac{vd}{v_f}$ { this was still good }

$\rightarrow \frac{1}{2}mv^2 = -\frac{GMm}{vd/v_f} + \frac{1}{2}mv_f^2 = -\frac{GMm}{vd} v_f + \frac{1}{2}mv_f^2$

$\rightarrow v_f^2 - \frac{2GM}{vd} v_f - v^2 = 0$

$\rightarrow v_f = \frac{2GM}{vd} \pm \sqrt{\frac{4G^2M^2}{v^2 d^2} + 4v^2} = \frac{GM}{vd} \pm \sqrt{\frac{G^2M^2}{v^2 d^2} + v^2}$

$v_f = \frac{GM}{vd} \left(1 \pm \sqrt{1 + \frac{v^4 d^2}{G^2 M^2}} \right)$; $r_f = \frac{vd}{v_f}$

Now units:

$\frac{GM}{d^2} = [m/s^2] \rightarrow \frac{G^2 M^2}{d^2} = [m^2/s^4 \cdot m^2] \rightarrow \frac{v^2 d^2}{G^2 M^2} = [1]$ ✓ $\frac{GM}{vd} = [m/s^2 \cdot m \cdot s/m] = [m/s]$ ✓

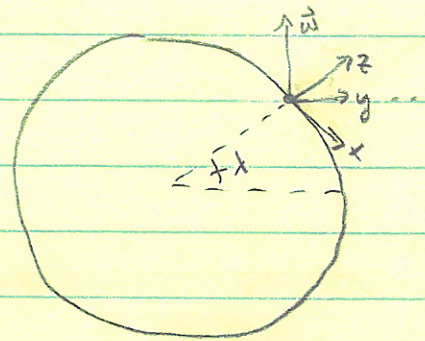
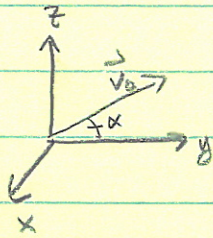
+ or - ? The $\sqrt{\quad}$ is always > 1 so for $v_f > 0$, it must be

(+) Use + root

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4)

$$\vec{a}_r = \vec{g} - 2\vec{\omega} \times \vec{v}_r$$



$$\vec{v}_r \sim v_0 \cos \alpha \hat{e}_y + (v_0 \sin \alpha - gt) \hat{e}_z$$

$$\vec{\omega} = -\omega \cos \lambda \hat{e}_x + \omega \sin \lambda \hat{e}_z$$

$$\vec{\omega} \times \vec{v}_r = -\omega v_0 \cos \lambda \cos \alpha \hat{e}_z + \omega \cos \lambda (v_0 \sin \alpha - gt) \hat{e}_y - \omega v_0 \cos \alpha \sin \lambda \hat{e}_x$$

$$\rightarrow \vec{a}_r \sim (-g - \omega v_0 \cos \lambda \cos \alpha) \hat{e}_z + (2\omega v_0 \cos \alpha \sin \lambda \hat{e}_x - 2\omega \cos \lambda (v_0 \sin \alpha - gt)) \hat{e}_y$$

$$a_{rz} \sim -g \rightarrow v_{rz} = v_0 \sin \alpha - gt; \quad z(t) = v_0 \sin \alpha t - \frac{1}{2}gt^2$$

$$t_f = t(z=0) = \frac{2v_0 \sin \alpha}{g}$$

$$d = x(t_f) = \omega v_0 \cos \alpha \sin \lambda t_f^2$$

$$\rightarrow d = \omega v_0 \cos \alpha \sin \lambda \left(\frac{4v_0^2 \sin^2 \alpha}{g^2} \right)$$

$$\rightarrow d = \frac{4v_0^3 \omega}{g^2} \cos \alpha \sin^2 \alpha \sin \lambda$$