Lecture 18 Friday Feb 21 Shadowitz: 9.1 Ohm's law

electro and magneto statics: no time dependence on $\vec{J}$ or ρ

$$\vec{J} = \rho \vec{v} \qquad \vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

$$\frac{C}{m^3} \quad \frac{m}{s} \qquad \frac{C}{s}\frac{1}{m^2} \qquad \int \vec{\nabla} \cdot \vec{J}\, d\tau = -\frac{\partial}{\partial t} \overbrace{\int \rho\, d\tau}^{Q_{enclosed}}$$

Conservation of charge

$$\oint \vec{J} \cdot da = -\frac{\partial}{\partial t} Q_{enclosed}$$

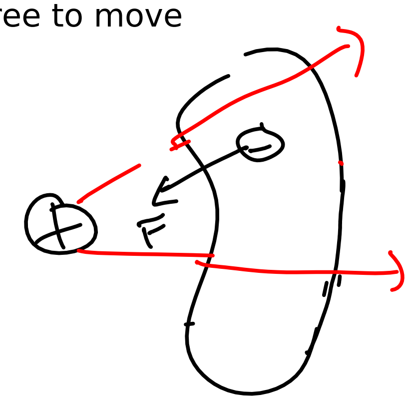
Ohm's law

$$\vec{J} = \sigma \vec{f}$$

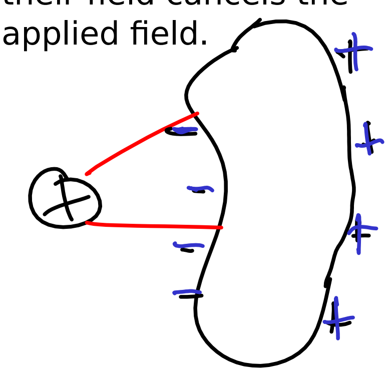
force/charge

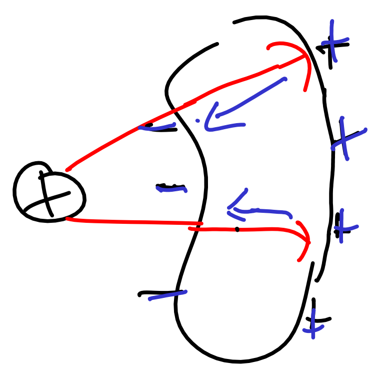
depends on material

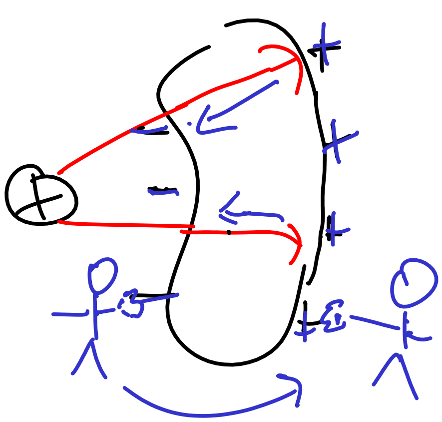
charge appears and E goes through the conductor causing a force on charges that are free to move



Charges move but their motion is damped. They come to rest so that their field cancels the applied field.







I pick up the charges and move them to the other side of the conductor causing a steady current to flow (acting as a battery)

This is our model for how charges in material behaves.

Questions:

-(informational) What is the force/charge?

It is whatever moves the charge (tweezers, gravity, etc.)

$$\vec{F} = q\vec{E} + q\vec{v}\times\vec{B} \simeq q\vec{E} \text{ for small } v$$

So Ohm's law becomes $\boxed{\vec{J} = \sigma \vec{E}}$

$$\vec{F} = q\vec{E} \Rightarrow \vec{E} = \frac{\overrightarrow{\text{Force}}}{\text{charge}}$$

-(causal) How is this law a consequence of the way charges behave microscopically?

See section 9.1 of Shadowitz

↓ more

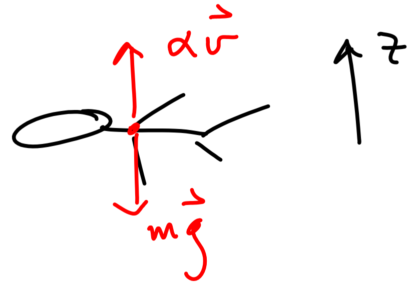
-(causal) Couldn't $\vec{J}$ go in a direction not parallel to $\vec{E}$?

In some materials yes. Then the math requires σ to be a matrix

$$\vec{J} = \sigma \vec{E}$$

-(analagous) What other physical phenomina obey similar relationships?

Skydiver falling at constant speed.



$$\Sigma F_z = \alpha v - mg = m a_z = 0$$

$$\frac{\alpha}{m} v = g$$

force/mass

Go to the InkSurvey site and answer the question about writing an analogous relationship to Ohm's law for many such skydivers of constant density moving at the same speed.

Also answer the question about what happens to these skydivers if g changes.

Also answer the question about what power is delivered to these skydivers by gravity.

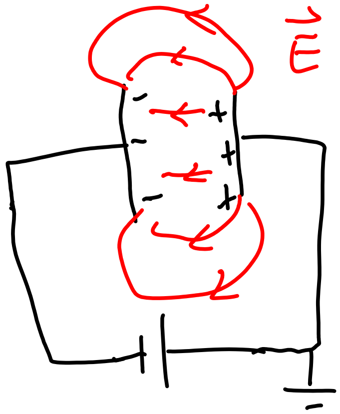
Questions:
(causal) What are the consequences of conservation of charge and Ohm's law?

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t} = 0 \qquad \text{statics}$$

$$\vec{J} = \sigma \vec{E}$$

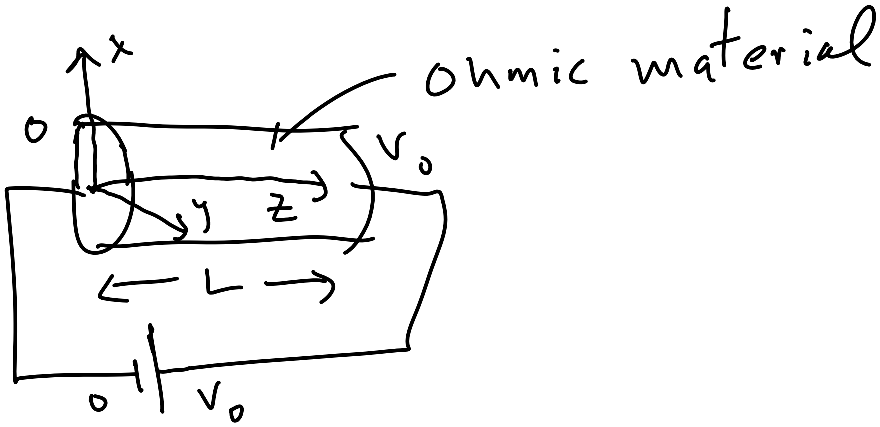
$$\vec{\nabla} \cdot \sigma \vec{E} = \sigma \vec{\nabla} \cdot \vec{E} = 0 \qquad \text{LAPLACE'S EQN.}$$

Example:



Field lines for a vacuum capacitor

What is E in the following cylindrical conductor?



Questions:

-(congruous) How do I calculate the electric field inside the conductor using Laplace's eqn?

-(congruous) I know Laplace's eqn applies but what are the boundary conditions needed to set up the relaxation method?

-(congruous) I know Laplace's eqn applies but how do I apply the boundary conditions to solve the problem analytically?

-(analogy) How does this relate to the flow of thermal energy which also obeys Laplace's eqn?

-(analogy) How does this relate to the flow of fluids?

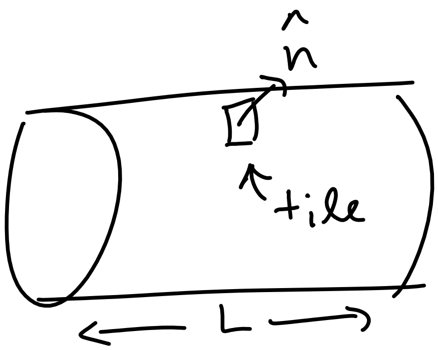
Boundary conditions:

$$\vec{J} = \rho \vec{v}$$

Ohm's law $\vec{J} = \sigma \vec{E}$

We assume that the charges do not leak out of the surface of the cylinder.

This gives us a condition on the electric field perpendicular to the cylinder surface.



$$\vec{J} \cdot \hat{n} = 0$$

but $\vec{J} = \sigma \vec{E}$ so

$$\vec{E} \cdot \hat{n} = 0$$

$\hat{n}$ is in $\hat{r}$ direction

and $$\vec{E} = -\vec{\nabla} V = -\left(\frac{\partial V}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial V}{\partial \theta} \hat{\theta} + \frac{\partial V}{\partial z} \hat{z} \right)$$ In the cylindrical coord system

This condition on E is also a condition on V at the boundary.

It turns out that if V or $\frac{\partial V}{\partial n}$ is specified at all surfaces then the solution is uniquely determined.

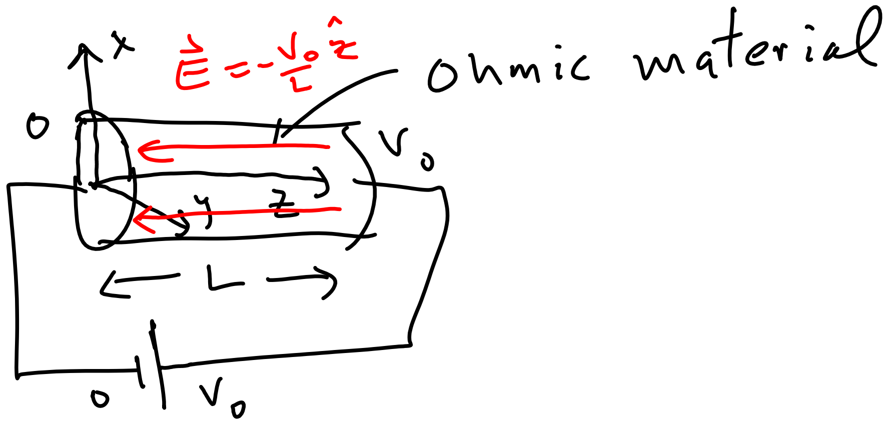
We know V at the two end caps and $\frac{\partial V}{\partial n}$ on the body of the cylinder so any solution that satisfies Laplace's eqn and
these bndry condition is THE soln.

We can guess a solution that satisfies Laplace's eqn and the bndry conditions:

$$V = \frac{V_0 z}{L} \quad ; \quad \frac{\partial V}{\partial n} = \frac{\partial V}{\partial r} = 0$$

$$\nabla^2 V = \underbrace{\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial V}{\partial r} \right)}_{=0} + \underbrace{\frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2}}_{=0} + \underbrace{\frac{\partial^2 V}{\partial z^2}}_{=0}$$ In cylindrical coords

$$\vec{E} = -\vec{\nabla} V = -\left(\underbrace{\frac{\partial V}{\partial r}}_{=0}\hat{r} + \frac{1}{r}\underbrace{\frac{\partial V}{\partial \theta}}_{=0}\hat{\theta} + \frac{\partial V}{\partial z}\hat{z}\right) = -\frac{V_0}{L}\hat{z}$$



Notice how simple the ohmic solution is in comparison with the free space capacitor solution for the electric field. Both are solutions to Laplace's equation but have different boundary conditions.