Let's talk about EM waves in matter

(i) $\vec{\nabla}\cdot\vec{D} = \emptyset$ (iii) $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$

(ii) $\vec{\nabla}\cdot\vec{B} = \emptyset$ (iv) $\vec{\nabla}\times\vec{H} = \frac{\partial \vec{D}}{\partial t}$

Assuming linear materials

$\vec{D} = \epsilon\vec{E}$; $\vec{H} = \frac{1}{\mu}\vec{B}$ $\epsilon = \epsilon_r\epsilon_0$ $\mu = \mu_r\mu_0$

Eg: glass $\epsilon_r \simeq 2.25$

(i) $\vec{\nabla}\cdot\vec{E} = \emptyset$ {only if ϵ is constant, eg. in glass}

(ii) $\vec{\nabla}\cdot\vec{B} = \emptyset$

(iii) $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$

(iv) $= \vec{\nabla}\times\vec{B} = \mu\epsilon\frac{\partial \vec{E}}{\partial t}$ { " }

Looks 'just like Maxwell's eqns in space

w/ $\epsilon_0 \to \epsilon$, $\mu_0 \to \mu$ $\Rightarrow$ $v_{light} = \frac{1}{\sqrt{\mu\epsilon}}$

If we define Index of refraction:

$n \equiv \sqrt{\frac{\mu\epsilon}{\mu_0\epsilon_0}}$ $\Rightarrow$ $v = \frac{c}{n}$ Typically, $\mu \simeq \mu_0$

$n = \sqrt{\epsilon_r}$

$u_{em} = \frac{1}{2}\left(\epsilon E^2 + \frac{1}{\mu}B^2\right)$ $\vec{S} = \frac{1}{\mu}\vec{E}\times\vec{B}$

Monochromatic plane waves!

$B = \frac{E}{v} = \frac{nE}{c}$; $\frac{\omega}{k} = v$

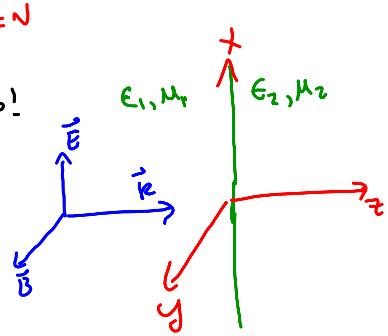
Boundary conditions!

(i) $\epsilon_1 E_1^{\perp} = \epsilon_2 E_2^{\perp}$

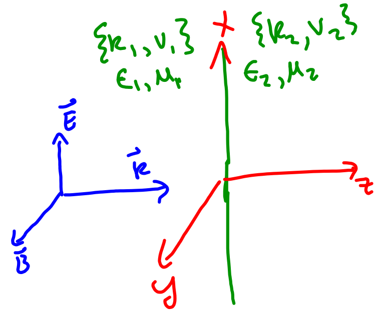
(ii) $B_1^{\perp} = B_2^{\perp}$

(iii) $\vec{E}_1^{\parallel} = \vec{E}_2^{\parallel}$

(iv) $\frac{1}{\mu_1}\vec{B}_1^{\parallel} = \frac{1}{\mu_2}\vec{B}_2^{\parallel}$



Let's solve for reflection/transmission for a monochromatic plane impinging on an interface at normal incidence



incident field

$$\vec{E}_I = \tilde{E}_{0I}\, e^{i(k_1 z - \omega t)}\,\hat{x}$$

$$\vec{B}_I = \frac{1}{v_1}\tilde{E}_{0I}\, e^{i(k_1 z - \omega t)}\,\hat{y}$$

Transmitted field

$$\vec{E}_T = \tilde{E}_{0T}\, e^{i(k_2 z - \omega t)}\,\hat{x}$$

$$\vec{B}_T = \frac{1}{v_2}\tilde{E}_{0T}\, e^{i(k_2 z - \omega t)}\,\hat{y}$$

Reflected field

$$\vec{E}_R = \tilde{E}_{0R}\, e^{i(-k_1 z - \omega t)}\,\hat{x}$$

$$\vec{B}_R = -\frac{1}{v_1}\tilde{E}_{0R}\, e^{i(-k_1 z - \omega t)}\,\hat{y}$$

Solve for $\tilde{E}_{0T}$, $\tilde{E}_{0R}$ in terms of $\tilde{E}_{0I}$ and other constants. Define $\beta = \frac{\mu_1 v_1}{\mu_2 v_2}$

Answer: $\tilde{E}_{0R} = \left(\frac{1-\beta}{1+\beta}\right)\tilde{E}_{0I}$; $\tilde{E}_{0T} = \left(\frac{2}{1+\beta}\right)\tilde{E}_{0I}$

Intensity $= I = \frac{1}{2}\epsilon v E_0^2 = \frac{1}{2}\sqrt{\frac{\epsilon}{\mu}}\,E_0^2$ {plane wave}

Reflection coef.: $R = \frac{I_R}{I_I}$

Transmission coef.: $T = \frac{I_T}{I_I}$

Solve for R, T for normal inc.

show $R + T = 1$ $\{ I_I = I_R + I_T \}$

Rules for reflection at a planar interface.

General Rules for any wave:

① $\vec{k}_I$, $\vec{k}_R$, $\vec{k}_T$, and normal vector to the interface plane all lie in the same plane.

② $\theta_I = \theta_R$

③ $\frac{1}{v_1}\sin\theta_I = \frac{1}{v_2}\sin\theta_T$ $\{n_1\sin\theta_I = n_2\sin\theta_T\}$

E & M

$$\beta = \frac{\mu_1 n_2}{\mu_2 n_1} = \frac{\mu_1 v_1}{\mu_2 v_2};\quad \alpha = \frac{\cos(\theta_T)}{\cos(\theta_I)}$$

TE:

$$\tilde{E}_{0R} = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)\tilde{E}_{0I};\quad \tilde{E}_{0T} = \left(\frac{2}{\alpha+\beta}\right)\tilde{E}_{0I}$$

$$R = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)^2 \qquad T = \alpha\beta\left(\frac{2}{\alpha+\beta}\right)^2$$

TM:

$$\tilde{E}_{0R} = \left(\frac{1-\alpha\beta}{1+\alpha\beta}\right)\tilde{E}_{0I};\quad \tilde{E}_{0T} = \left(\frac{2}{1+\alpha\beta}\right)\tilde{E}_{0I}$$

$$R = \left(\frac{1-\alpha\beta}{1+\alpha\beta}\right)^2;\quad T = \alpha\beta\left(\frac{2}{1+\alpha\beta}\right)^2$$