Maxwell-Boltzmann distribution function

The M-B distribution function can be written in terms of energy as

f(E) = A exp(- E/kT)

For a free gas (particle energy is only kinetic), the distribution function can be written in terms of velocity:

$f(v) = A \exp\left(-\frac{1}{2} m v^2 / \mathrm{kT}\right)$

Here, n(r) is the number density, which can in general vary with position. The constant A is determined by normalization. We will treat this as a probability distribution function normalized so that

$\int f(v)\, d^3 v = \int A \exp\left(-\frac{1}{2} m v^2 / \mathrm{kT}\right) d^3 v = 1$, so

$\mathrm{A} = \left(\int \exp\left(-\frac{1}{2} m v^2 / \mathrm{kT}\right) d^3 v\right)^{-1}$

We make use of the common integral of a Gaussian:

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In[1]:= Integrate[Exp[-z^2], {z, -∞, ∞}]

Out[1]= √π
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Let $x^2 = \frac{1}{2} m v_x{}^2 / k\,T$, so dx =dv $\sqrt{m / 2\,k\,T}$

$\int \exp\left(-\frac{1}{2} m v^2 / \mathrm{kT}\right) d^3 v = \int \exp\left(-\frac{1}{2} m v_x{}^2 / \mathrm{kT}\right) d v_x \int \exp\left(-\frac{1}{2} m v_y{}^2 / \mathrm{kT}\right) d v_y \int \exp\left(-\frac{1}{2} m v_z{}^2 / \mathrm{kT}\right) d v_z$

$= \left(\frac{2\,\pi\,k\,T}{m}\right)^{3/2}$

we'll work with a situation where the particle density is constant, n(r) = n_0, so the normalized distribution function is

$f(v) = \left(\frac{m}{2\,\pi\,k\,T}\right)^{3/2} \exp\left(-\frac{1}{2} m v^2 / \mathrm{kT}\right)$

The interpretation of this function is that $f\left(v_x, v_y, v_z\right) \mathrm{dv}_x\, \mathrm{dv}_y\, \mathrm{dv}_z$ is the probability of finding a particle with a velocity in a narrow range of velocities centered on v. If we want to know the probablility of finding a particle with a z-component of velocity v_z (with whatever x and y components), we integrate over v_x and v_y:

$f(v_z)\, \mathrm{dv}_z = \left(\frac{m}{2\,\pi\,k\,T}\right)^{1/2} \exp\left(-\frac{1}{2} m v_z{}^2 / \mathrm{kT}\right) \mathrm{dv}_z$