MATH 348 - Advanced Engineering Mathematics February 12, 2008

Exam I - Review **Exam I**: February 22, 2008

Exam I will be held Friday the 22^{nd} in class. There will be no notecards or calculators. Exam I will test on Chapter 11 from the text. To prepare for the exam you should refer to the recommended problems on the syllabus and the homework assignments. The following is a list of concepts and methods which you should be familiar with. Also listed are the key equations from Chapter 11. It is assumed that the student has equations (1)-(4) and (9) memorized for the exam.

11.1-2 Fourier Series of Periodic Functions (Formulas (1)-(2))

From this section the student should understand:

- The concept of an orthogonal trigonometric system.
- The concept of representing periodic functions using trigonometric series.

From this section the student should be able to:

- Determine Fourier coefficients of a 2L-periodic function.
- Using the Fourier coefficients, write down the Fourier series representation of a 2L-periodic function.

11.3 Even and Odd Functions. Half-Range Expansions

From this section the student should understand:

- The algebraic and geometric properties of even and odd functions.
- The definite integral simplifications associated with even and odd functions.
- The Fourier series representations of functions with symmetry.

From this section the student should be able to:

- Simplify integrals based on symmetry of the integrand.
- Simplify Fourier series based on symmetry of the periodic function.
- Periodically extend functions whose domain is finite to get Half-Range series expansions.

11.4 Complex Fourier Series (Formulas (3)-(4))

From this section the student should understand:

- The connection between exponential functions and sine/cosine functions.
- The equivalence of the real Fourier series and the complex Fourier series.

From this section the student should be able to:

- Determine the complex Fourier coefficients of a 2L-periodic function.
- Using the complex Fourier coefficients, write down the complex Fourier series representation of a 2L-periodic function.
- Determine the equivalent real Fourier series representation of a function given the complex Fourier series.

11.7 Fourier Integral (Formulas (5)-(6))

From this section the student should understand:

- The connection between periodic functions and non-periodic functions.
- The relationship between the Fourier Series and the Fourier Integral.

- Simplifications of the Fourier Integral associated with non-periodic symmetric functions.

From this section the student should be able to:

- Determine the Fourier Integral representation of a non-periodic function.

11.8 Fourier Sine and Cosine Transforms (Formulas (7)-(8))

From this section the student should understand:

- The connection between the Fourier Integral representation of symmetric functions and the Fourier Sine/Cosine Transforms (both forward and inverse transforms).
- The connection between sine and cosine transforms and odd and even functions.

From this section the student should be able to:

- Given a function, find the sine/cosine transform.

11.9 The Fourier Transform (Formula (9))

From this section the student should understand:

- The connection between the Fourier Integral and the Fourier Transform.
- The concept of transform pairs.

From this section the student should be able to:

- Determine the Fourier Transform of a non-symmetric function.
- Determine the Fourier Transform of a function using transform pairs.

Important formulas: The following equations will be needed for the examination.

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{L}x\right) + b_n \sin\left(\frac{n\pi}{L}x\right) \tag{1}$$

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)dx, \qquad a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi}{L}x\right)dx, \qquad b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi}{L}x\right)dx. \tag{2}$$

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{\frac{in\pi}{L}x} \tag{3}$$

$$c_n = \frac{1}{2L}\int_{-L}^{L} f(x)e^{-\frac{in\pi}{L}x}dx \tag{4}$$

$$f(x) = \int_0^{\infty} [A(\omega)\cos(\omega x) + B(\omega)\sin(\omega x)]d\omega \tag{5}$$

$$A(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\cos(\omega x)dx, \qquad B(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\sin(\omega x)dx \tag{6}$$

$$\hat{f}_c(\omega) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x)\cos(\omega x)dx, \qquad f(x) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} \hat{f}_c(\omega)\cos(\omega x)d\omega \tag{7}$$

$$\hat{f}_s(\omega) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x)\sin(\omega x)dx, \qquad f(x) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} \hat{f}_s(\omega)\sin(\omega x)d\omega \tag{8}$$

$$\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{-i\omega x}dx \qquad f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \hat{f}(\omega)e^{i\omega x}d\omega \tag{9}$$