2 beam interference

- measure <u>intensity</u>; but <u>fields</u> add

intensity for arbitrary polarization:

$$I = \tfrac{1}{2}\varepsilon_0 c n\, \vec{E}\cdot\vec{E}^* = \tfrac{1}{2}\varepsilon_0 c n\left(E_x\cdot E_x^* + E_y E_y^* + E_z E_z^*\right)$$

combine two waves

<u>different polarization</u>

$$I = \tfrac{1}{2}\varepsilon_0 c n\,(E_1\hat{x} + E_2\hat{y})\cdot(E_1^*\hat{x} + E_2^*\hat{y})$$

$$= \tfrac{1}{2}\varepsilon_0 c n\left(|E_1|^2 + |E_2|^2\right) = I_1 + I_2$$

since intensities <u>add</u> no interference (no cross-terms)

<u>same polarization</u>

$$I = \tfrac{1}{2}\varepsilon_0 c n\,\left((E_1+E_2)\hat{x}\right)\cdot(E_1^*+E_2^*)\hat{x}$$

$$= I_1 + \underbrace{\tfrac{1}{2}\varepsilon_0 c n\cdot 2\,\mathrm{Re}(E_1E_2^*)}_{\text{cross term shows interference.}} + I_2$$

In general, orthogonal states of pol. don't interfere.

$\hat{x},\hat{y}$ or R, L circular.

Combine two equal plane waves w/ phase difference δ

$$E_{tot} = E_0 + E_0 e^{i\delta}$$

$$I_{tot} \propto |E_{tot}|^2 = |E_0|^2 |1 + e^{i\delta}|^2$$
$$= |E_0|^2 (1 + 1 + e^{i\delta} + e^{-i\delta})$$

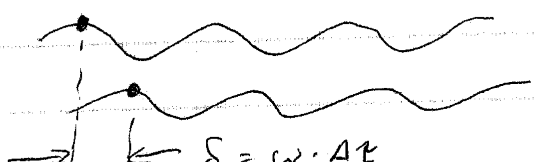
$$= |E_0|^2 (2 + 2\cos\delta)$$

when phase difference $\delta = m\pi$,
$|E_{tot}|^2 = 4|E_0|^2$ constructive interference

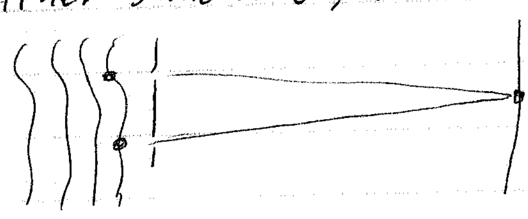
when phase diff $= \delta = m\pi + \pi/2$
$|E_{out}|^2 = 0$ destructive interf.

δ = phase difference: optical path diff., reflection phase shifts
$= \omega(t_1 - t_2)$ time diff
$= k_0(n_1 z_1 - n_2 z_2)$ optical path diff.

Ultimately, we compare a wave with another wave.
- amplitude split: compare wave w/ copy at later time



- wavefront split: compare diff't portions of wavefront
  - either same time, or diff't time



Temporal coherence time τ_c

incoherence is random phase

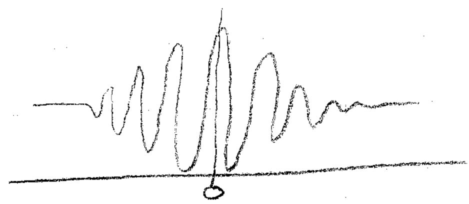
- single atom 

  collisions interrupt phase.

- multi-atom : initial phase is all random.

net effect: light wave keeps its coherence for an average time τ_c

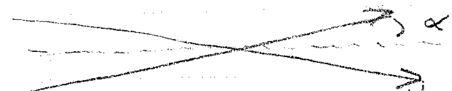
Measure τ_c : $I(\Delta\tau)$



Fringes wash out

reduced <u>contrast</u> or <u>visibility</u>. $\dfrac{I_{max} - I_{min}}{I_{max} + I_{min}}$

Misaligned Michelson: tilt mirrors



as before, $E = E_0 e^{ik_z z}\left(e^{ik_x x} + e^{-ik_x x}\right)$

$k_x = k_0 \sin\alpha$

$|E|^2 \sim |E_0|^2 \cos^2 k_x x$

if arm lengths are initially equal, space interference pattern follows <u>time</u> pattern.

α very small

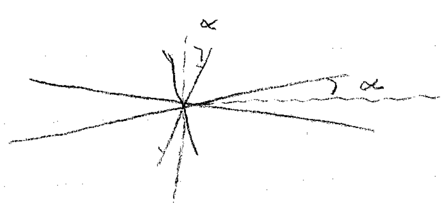
x period $\sim 2\pi / 2k_0 \sin\alpha$

$\Delta x = \lambda / 2\sin\alpha$

for $\Delta x \sim 100\,\mu m$ $\lambda \sim 0.5\,\mu m$

$\alpha \sim \lambda / 2\Delta x \approx 2.5\,mrad$

$= 0.14°$



$\tau(x) = \pm \dfrac{x \tan\alpha}{c}$

$\omega \Delta\tau(x) = \pm \dfrac{\omega}{c} x \tan\alpha$

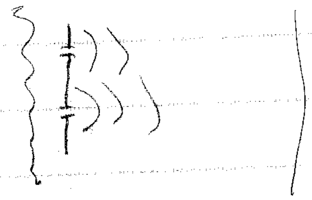
$E \sim E_0\left(e^{ikx\tan\alpha} + e^{-ikx\tan\alpha}\right)$

$\sim 2E_0 \cos(kx\tan\alpha)$

Spatial coherence length

sample wavefront across beam, compare

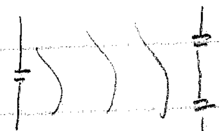
measure: vary separation of two point sources



extended sources - lamps, sun

have very small spatial coherence lengths

Either sample small portion of wavefront.

ex:  Young's

or spectrometer w/ narrow entrance slit.

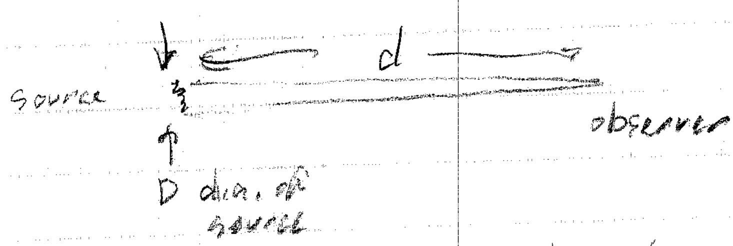
Or move very far away: starlight has high spatial coh.

- stellar interferometry: can measure size of star.

Van Cittert-Zernike theorem

spatial coherence area is

$$A_c = \frac{D^2\lambda^2}{\pi d^2} = \frac{\lambda^2}{\Delta\Omega}$$



wavefronts smooth out w/ distance.