Lecture 6

Note Title 1/23/2006

Types of problems you encounter

① Calculate V given $\vec{E}$

$$\Delta V = -\int \vec{E} \cdot d\vec{l} \qquad q\Delta V = \Delta PE = -W_{cons}$$

② calculate $\vec{E}$ given $V(x,y,z)$

$$-\int \vec{E} \cdot d\vec{l} = \int \vec{\nabla} V \cdot d\vec{l} = \int \left(\hat{x}\frac{\partial V}{\partial x} + \hat{y}\frac{\partial V}{\partial y} + \hat{z}\frac{\partial V}{\partial z} \right) \cdot$$

$\vec{E} = -\vec{\nabla} V$

$(dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z})$

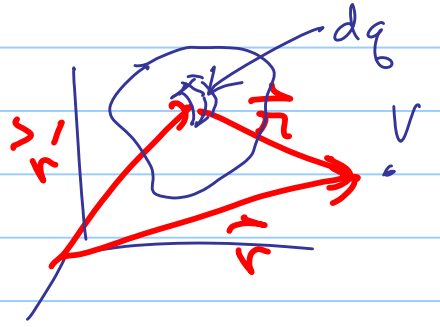
$$= \int_i^f \underbrace{\left(\frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz\right)}_{dV} = V_f - V_i$$

(3) Calculate V given $\rho(x,y,z)$

$\Delta V = -\int \vec{E}\cdot d\vec{l}$ point charge $\Delta V =$

$$\Delta V = -\int (\vec{E}_1 + \vec{E}_2 + \cdots)\cdot d\vec{l} = -\int \vec{E}_1\cdot d\vec{l} - \int \vec{E}_2\cdot d\vec{l} + \cdots$$

$$= \frac{q_1}{4\pi\epsilon_0|\vec{r}-\vec{r}_1'|} + \frac{q_2}{4\pi\epsilon_0|\vec{r}-\vec{r}_2'|} + \cdots$$



Charge distribution $\Sigma \rightarrow \int$

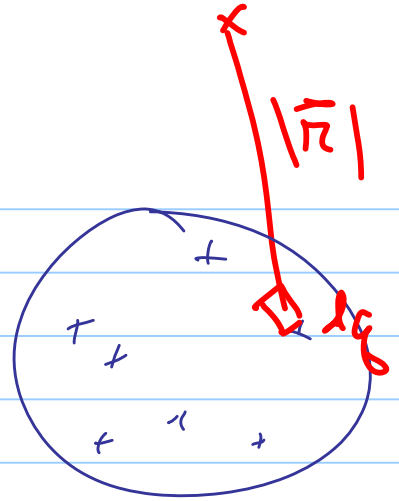
$$V = \int \frac{1}{4\pi\epsilon_0} \frac{dq}{|\vec{r} - \vec{r}'|}$$

(4) Calculate ρ given V or E

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enl}}{\epsilon_0} = \frac{\int \rho \, d\tau}{\epsilon_0} \Rightarrow \boxed{\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0}$$

$\vec{E} = -\vec{\nabla} V$

$$\nabla^2 V = -\rho/\epsilon_0$$



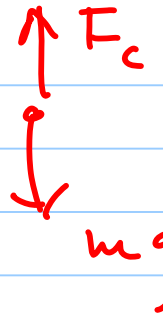
✓ $\Delta V = -\int \vec{E} \cdot d\vec{l}$

$\Delta V = \int \frac{k\, dq}{r}$

Solve 3-D PDE

$\nabla^2 V = -\rho/\epsilon_0$

Hard



$Q \leftarrow$ Head

$-Q \leftarrow$ balloon

$mg = k\frac{Q^2}{r^2}$

$Q \approx 10^{10}$ electrons