Note Title 7/25/2006

cons of charge

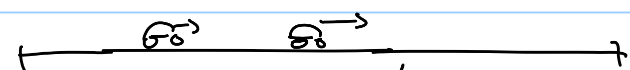
3.) $\vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$ $\underline{3-D}$

div th $\quad -\frac{\partial}{\partial t}\int \rho\, d\tau$

$\int \vec{\nabla}\cdot\vec{J}\, d\tau = \underbrace{\oint \vec{J}\cdot d\vec{a}}_{\text{flow of charge out of a surface}} = -\frac{\partial}{\partial t} Q_{encl}$ ← charge within surface decreases

No source of charge

1-D example



x_i $\qquad$ x_f

$J = \lambda v$ ← speed

λ: cars length

$\int \vec{\nabla}\cdot\vec{J}\, d\tau = \int\left(\frac{\partial}{\partial x}J_x + \frac{\partial}{\partial y}J_y + \cdots\right) d\tau$

↓ 1-D

$\int \frac{\partial J_x}{\partial x}\, dx = -\frac{\partial}{\partial t}\int \lambda\, dx$

$\int dJ_x = J_{x,f} - J_{x,i} = -\frac{\partial}{\partial t}\underbrace{\int \lambda\, dx}_{\#\text{ cars within } x_f - x_i}$

$J = \lambda v$ ← speed

↑ car density

4.) Retarded potentials $\Rightarrow$ $\vec{B} = \vec{\nabla} \times \vec{A}$ $\quad \vec{E} =$

$$\vec{A}(r,t) = \int \frac{\vec{J}(r', t_r)}{\mathscr{r}} \, d\tau'$$

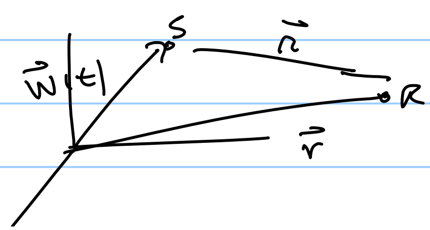
$ct = \mathscr{r} = 10^3 \text{ m}$ $\qquad t = \frac{10^3}{3 \times 10^8} = \frac{1}{3} \times 10^{-5}$

5.) $\vec{\nabla} |\vec{r} - \vec{r}'| = \vec{\nabla} \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$

$$\hat{x} \frac{\partial}{\partial x} \sqrt{} + \hat{y} \frac{\partial}{\partial y} \sqrt{} + \hat{z} \frac{\partial}{\partial z} \sqrt{} = \hat{x} \frac{1}{\cancel{2}} \frac{\cancel{2}(x-x')}{\sqrt{}} + \hat{y} \cdots$$

$$= \frac{\hat{x}(x-x') + \hat{y}(y-y') + \hat{z}(z-z')}{\sqrt{}} = \frac{\vec{\mathscr{r}}}{|\vec{\mathscr{r}}|} = \hat{\mathscr{r}}$$

6.)



flash goes off at source $w(t_r)$. Distance light travels is $|\vec{\mathscr{r}}| = |\vec{r} - \vec{w}(t_r)|$ and this distance is speed

time time $(t - t_r)$ $\quad$ $|\vec{r} - w(t_r)| = c(t - t_r)$

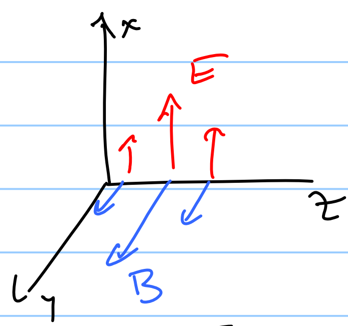
now $\quad$ when flash went off $\quad$ $x_0 + vt$

Not the same as when the source & rec. are stationary $\left(t_r = \underbrace{t - \frac{r}{c}}\right)$

8.) $$\frac{dr}{dt} = \cancel{\frac{\partial r}{\partial t}} + (\vec{v} \cdot \vec{\nabla})\vec{r} = \left(v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z}\right) x\hat{x} + \cdots$$

0 $\quad$ fix x, y, z $\quad$ $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$

11.)



$$T_{ij} = \epsilon_0\left(E_i E_j - \frac{1}{2}\delta_{ij} E^2\right) + \frac{1}{\mu_0}\left(B_i B_j - \frac{1}{2}\delta_{ij} B^2\right)$$

E has only $\hat{x}$ component

B " " $\hat{y}$ "

$i \neq j$ $\quad T_{ij} = \_\_ E_x E_y + \_\_ B_x B_y = 0$

$i = 1, 2, 3 : x, y, z$

$\vec{F} = q\vec{v} \times \vec{B} + q\vec{E}$

$$T_{xx} = \epsilon_0\left(E_x^2 - \tfrac{1}{2}E^2\right) + \frac{1}{\mu_0}\left(B_x^{2 \to 0} - \tfrac{1}{2}B^2\right)$$

$$B^2 = \frac{E^2}{c^2} = \frac{E^2}{\frac{1}{\mu_0\epsilon_0}}$$

$$= \epsilon_0 \frac{1}{2}E^2 - \frac{1}{\mu_0}\frac{1}{2}B^2$$

$$= \epsilon_0 \frac{1}{2}E^2 - \frac{1}{\cancel{\mu_0}}\frac{1}{2}E^2\cancel{\mu_0}\epsilon_0 = 0$$

$$T_{yy} = 0$$

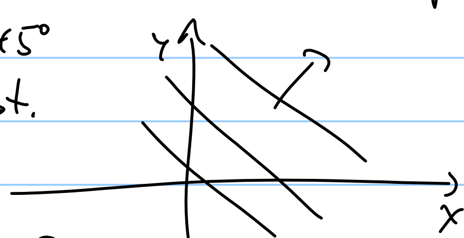
energy density

$$T_{zz} = -\frac{1}{2}\left(\epsilon_0 E^2 + \frac{B^2}{\mu_0}\right) = -u$$

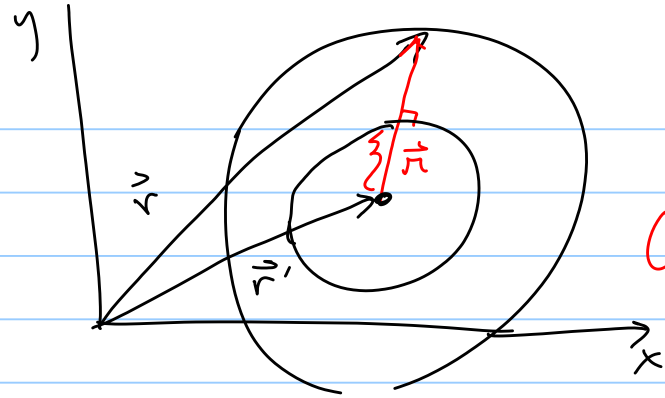
$\int \overleftrightarrow{T}\cdot d\vec{a}$ rate at which momentum crosses area

---

Animate with Mathematica a plane wave moving a 45° in a contour plot.



ContourPlot[



$$\vec{k} = \frac{2\pi}{\lambda}\hat{\imath}$$

$$\varphi = \vec{k}\cdot\vec{\imath} = \frac{2\pi}{\lambda}\hat{\imath}\cdot\vec{\imath}$$

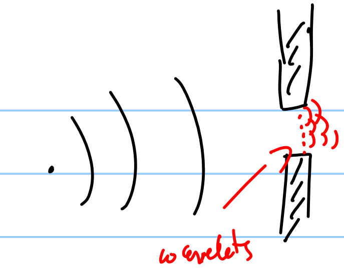
$$= \frac{2\pi}{\lambda}\frac{\vec{\imath}}{|\vec{\imath}|}\cdot\vec{\imath} = \frac{2\pi}{\lambda}\frac{\imath^2}{\imath}$$

$$\vec{\imath} = \vec{r} - \vec{r}\,' = (x-x')\hat{x} + (y-y')\hat{y}$$
$$+(z-z')\hat{z}$$

$$\varphi = \frac{2\pi}{\lambda}|\vec{\imath}|$$

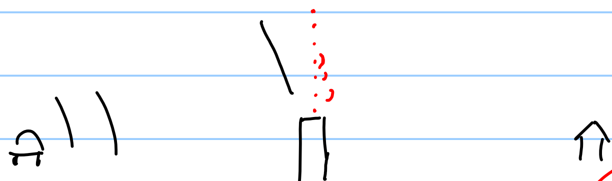
$$\cos(\vec{k}\cdot\vec{\imath} - \omega t)$$

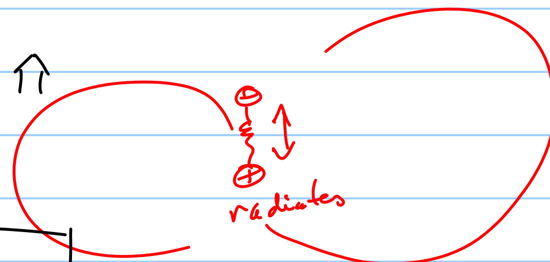
Wave eqn: PDE can be by Huygens Principle

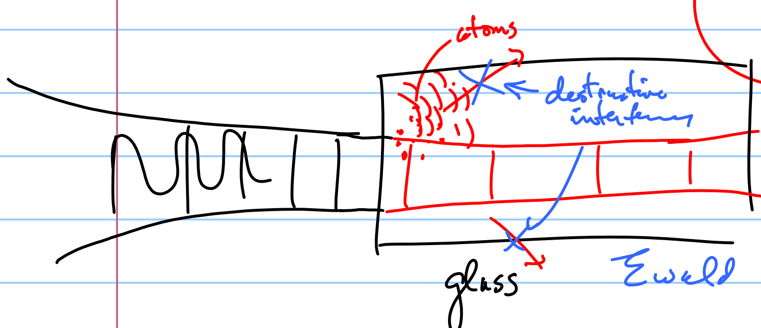


× amplitude = sum of wave amplitudes from wavelets

$\cos(\vec{k}\cdot\vec{r} - \omega t)$



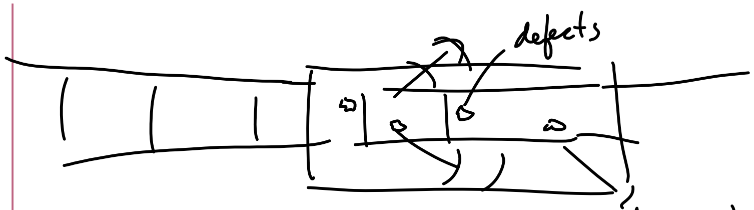




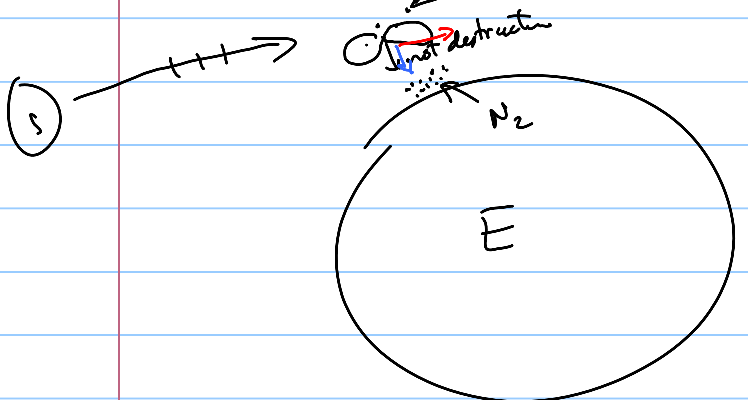
Ewald Oseen theorem (spacing of atoms less than λ)

macroscopic ME $\Rightarrow$ $c \to \frac{c}{n}$ $\qquad \lambda \to \frac{\lambda}{n}$

microscopic



density fluctuations whose size ≥ λ



spacing of N_2 near surface is $\ll \lambda$