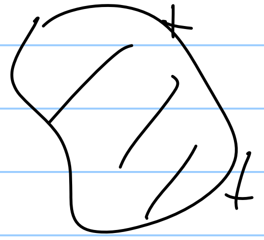
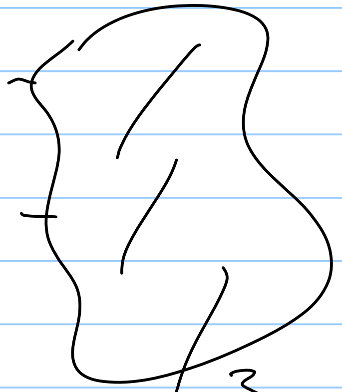




$q = CV$

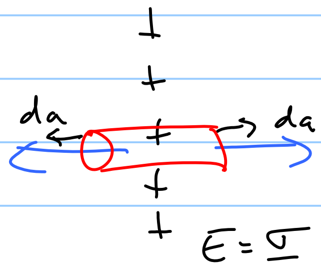
$$\int dW = \int dq\, V = \int_0^{Q_f} dq\, \overset{V}{\overset{\downarrow}{\frac{q}{C}}} = \frac{1}{C}\,\frac{1}{2}\,Q_f^2$$

$$W = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{C^2V^2}{C} = \frac{1}{2}CV^2$$

$$\frac{1}{2}\,10^2 = 50\text{ J}$$

10 V on 1 F cap

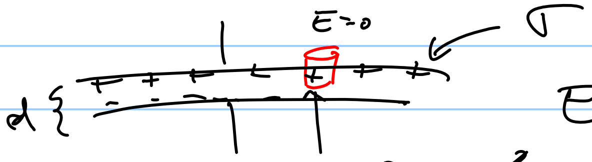
$$\boxed{\begin{aligned} &✓\ W_{me} = \int dq\, V \\ &✓\ W_{me} = \frac{1}{2}\int dq\, V \end{aligned}}$$



$E = \frac{\sigma}{2\epsilon_0}$

$$\overbrace{E\pi r^2 + E\pi r^2}^{\text{flux}} = \frac{\sigma \pi r^2}{\epsilon_0}$$

Parallel plate cap



$E = \frac{\sigma}{\epsilon_0}$

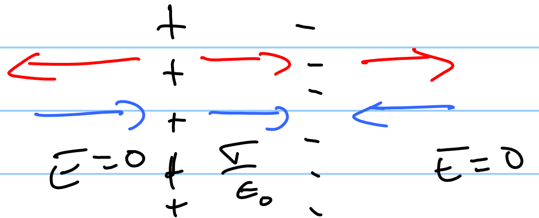
$$C = \frac{Q}{V} = \frac{\cancel{\sigma} A}{\frac{\cancel{\sigma}}{\epsilon_0} d} = \epsilon_0 \frac{A}{d}$$

$$E\pi r^2 = \frac{\sigma \cancel{\pi r^2}}{\epsilon_0}$$

$$\underline{\left|\Delta V\right| = \left|-\int \vec{E}\cdot d\vec{r}\right| = \frac{\sigma}{\epsilon_0}d}$$

$$W_{me} = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{\sigma^2 A^2 d}{\epsilon_0 A}$$

$$\boxed{= \frac{1}{2}\sigma^2 A d/\epsilon_0}$$



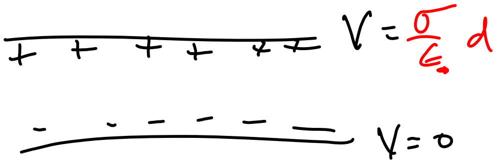
$$W_{me} = \frac{1}{2}\int dq\, V$$

$$= \frac{1}{2}\int_{\text{lower}} dq\, \cancel{V} + \frac{1}{2}\int_{\text{upper}} dq\, V$$

$$\xrightarrow{} 0$$

const

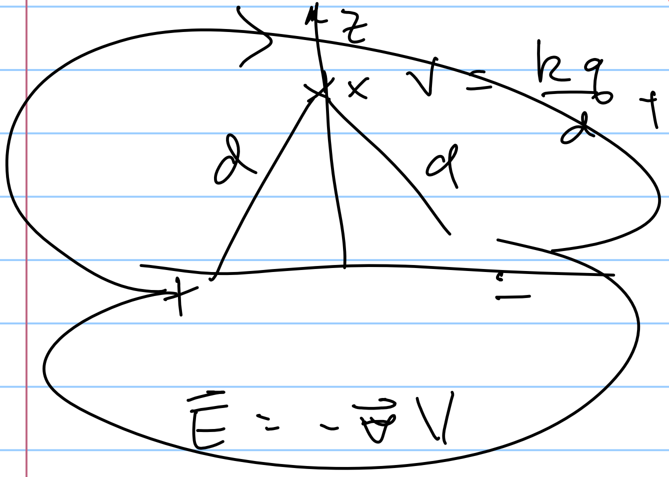
$$= \frac{1}{2} V \int dq = \frac{1}{2}\frac{\sigma}{\epsilon_0} d\, \sigma A = \frac{1}{2}\frac{\sigma^2}{\epsilon_0} A d$$



$$W_{me} = \int \frac{1}{2}\epsilon_0 E^2 d\tau = \frac{1}{2}\epsilon_0 E^2 A d = \frac{1}{2}\epsilon_0 \frac{\sigma^2}{\epsilon_0^2} A d$$

const

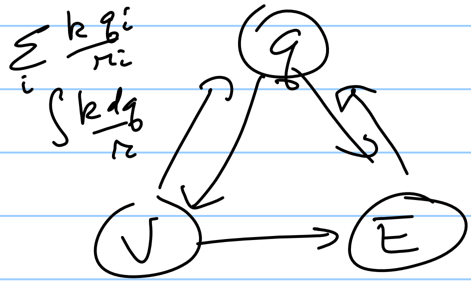
all space but E = 0 outside cap



$$V = \frac{kq}{d} + \frac{k(-q)}{d} = 0$$

$$\vec{E} = -\vec{\nabla} V$$

gives $V(x=0, y=0, z=h)$



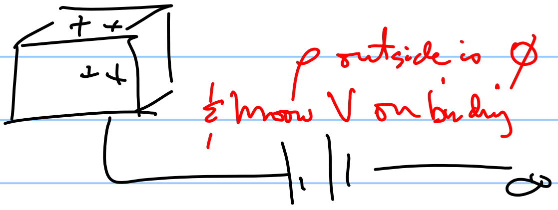
$$\vec{\nabla} V = \hat{x}\frac{\partial V}{\partial x} + \hat{y}\frac{\partial V}{\partial y} + \hat{z}\frac{\partial V}{\partial z}$$

Summation problem: given ρ find E & V

Bndry value value problem

given V find ρ

$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0}$$



ρ outside is 0 & know V on bndry

$$\int \overset{\downarrow}{\underline{\vec{\nabla}\cdot\vec{E}}}\, d\tau = \int \boxed{\frac{\rho}{\epsilon_0}}\, d\tau \qquad \vec{\nabla}\cdot\underset{\underset{-\vec{\nabla}V}{\parallel}}{\vec{E}} = \rho/\epsilon_0$$

$$\nabla^2 V = -\rho/\epsilon_0$$ Poisson's eqn

given V on boundary need to find V everywhere

Solve $\nabla^2 V = 0$ outside to get V

near boundary

$$E_{\perp} = \frac{\sigma}{\epsilon_0}$$

$$\Big\downarrow$$

$$\left(-\vec{\nabla}V\right)_{\leftarrow\,\text{perp}} = \frac{\sigma}{\epsilon_0}$$