Tilted window: ray propagation

- Calculate phase shift caused by the **insertion** of the window into an interferometer.
- Ray optics:
  - Add up optical path for each segment
  - Subtract optical path w/o window

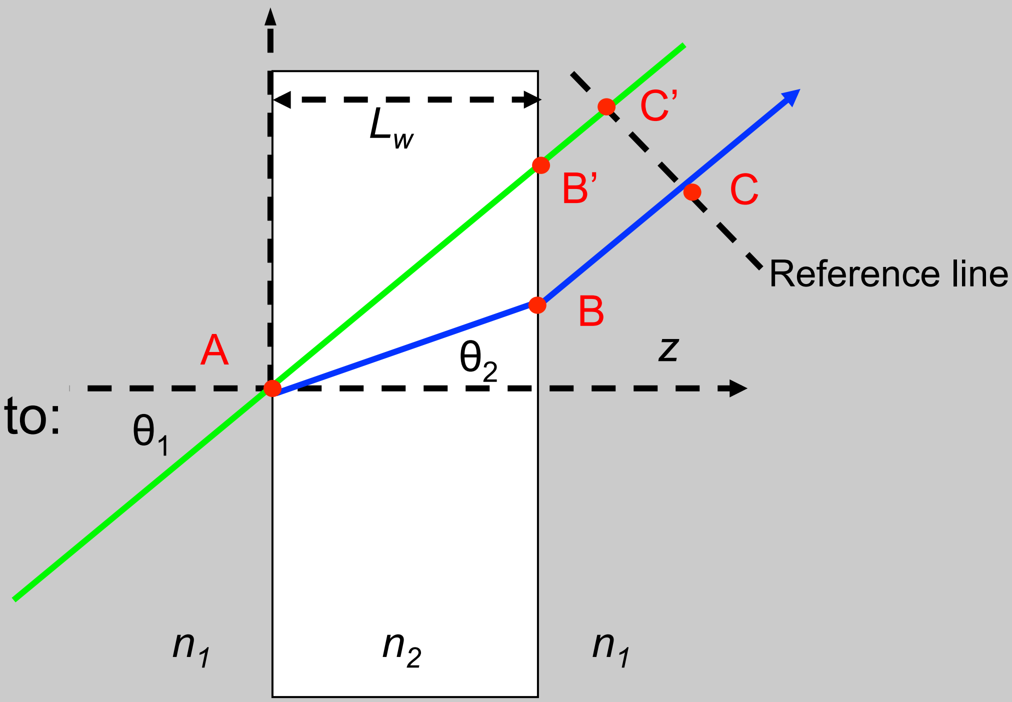
$$\Delta d = nL_{AB} + L_{BC} - L_{AB'} - L_{B'C'}$$

$$L_{AB} = \frac{L_w}{\cos\theta_2} \quad L_{AB'} = \frac{L_w}{\cos\theta_1}$$

$$L_{BC} = L_{B'C'} + L_{BB'} \sin\theta_1$$

  - Use Snell's Law to reduce to:

$$\Delta d = n\,L_w \cos\theta_2 - L_w \cos\theta_1$$



Tilted window: wave propagation

- Write expression for tilted plane wave

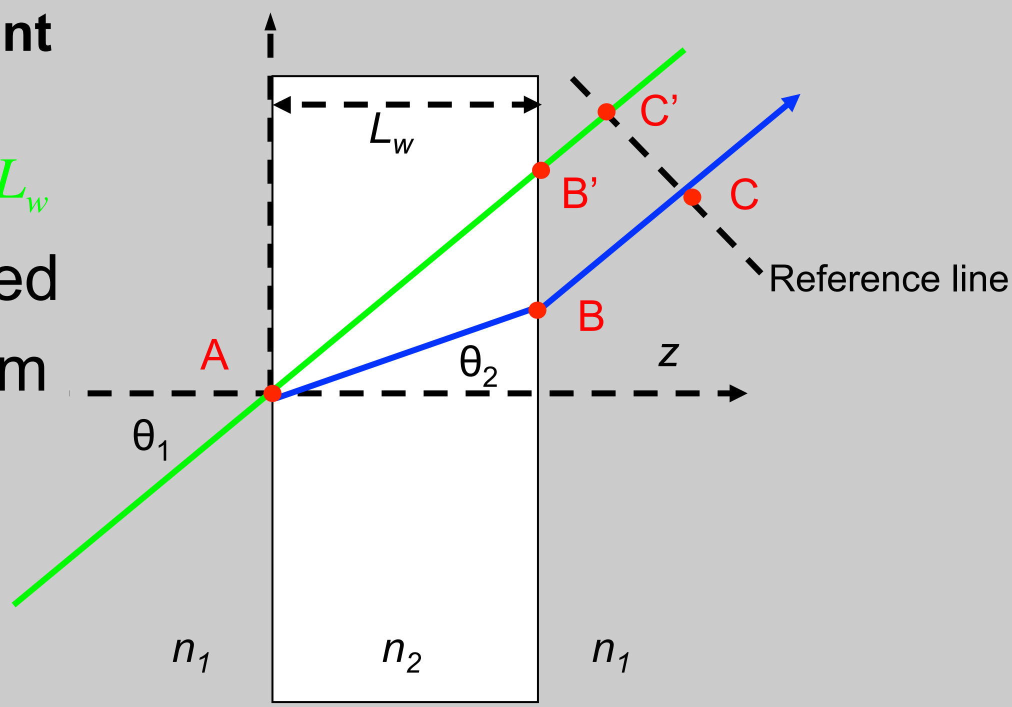
$$E(x,z) = E_0 \exp\left[i\left(k_x x + k_z z\right)\right] = E_0 \exp\left[i\frac{\omega}{c}n\left(x\sin\theta_2 + z\cos\theta_2\right)\right]$$

- Snell's Law: phase across surfaces is conserved

$$k_x x = \frac{\omega}{c} n \sin\theta$$ **is constant**

$$\Delta\phi = \left(k_2\cos\theta_2\right)L_w - \left(k_1\cos\theta_1\right)L_w$$

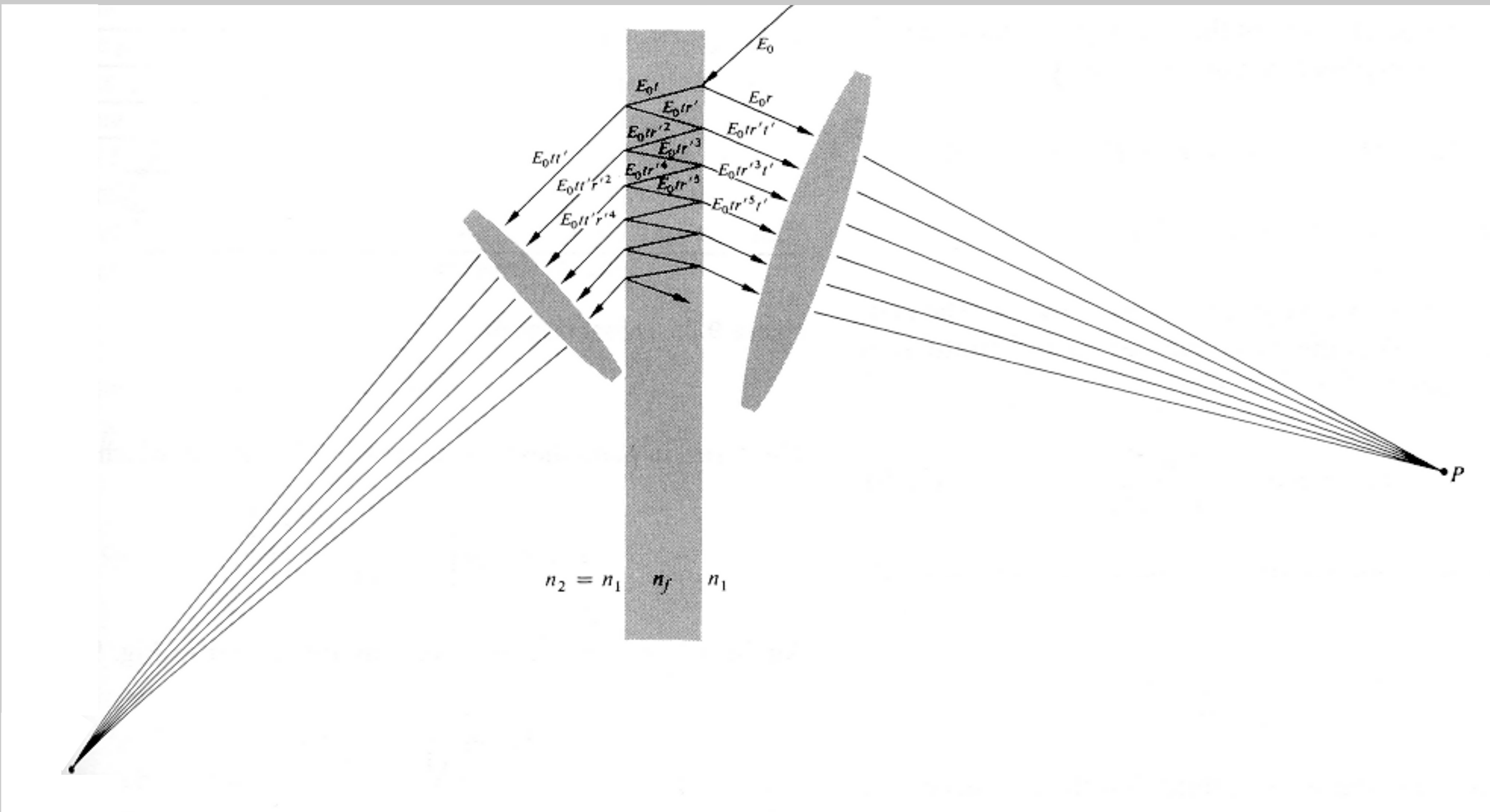
- This approach can be used to calculate phase of prism pairs and grating pairs



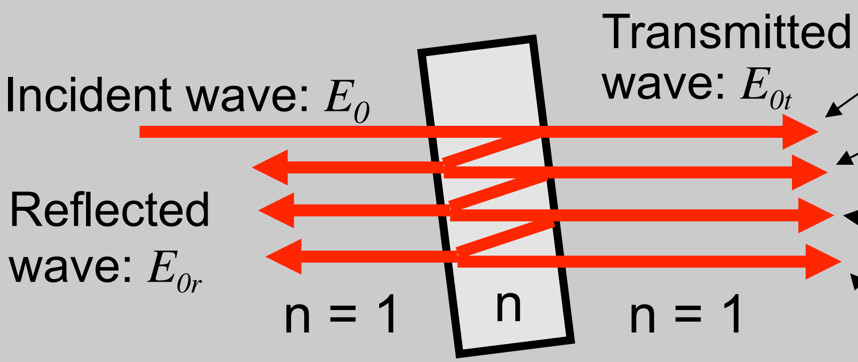
Multiple-beam interference: The Fabry-Perot Interferometer or Etalon

A Fabry-Perot interferometer is a pair of **parallel** surfaces that reflect beams back and forth. An etalon is a type of Fabry-Perot interferometer, and is a piece of glass with parallel sides.

The transmitted wave is an infinite series of multiply reflected beams.



Multiple-beam interference: general formulation



$$t\,t'\,e^{i\delta/2}\,E_0$$

$$t\,t'\,e^{i\delta/2}\left(r'\right)^2 e^{i\delta}\,E_0$$

$$t\,t'\,e^{i\delta/2}\left(\left(r'\right)^2 e^{i\delta}\right)^2 E_0$$

$$t\,t'\,e^{i\delta/2}\left(\left(r'\right)^2 e^{i\delta}\right)^3 E_0$$

r, t **= reflection, transmission coefficients from air to glass**
r', t' **= " " " from glass to air**

δ = round-trip phase delay inside medium = $k_0(2\ n\ L \cos\theta_t)$

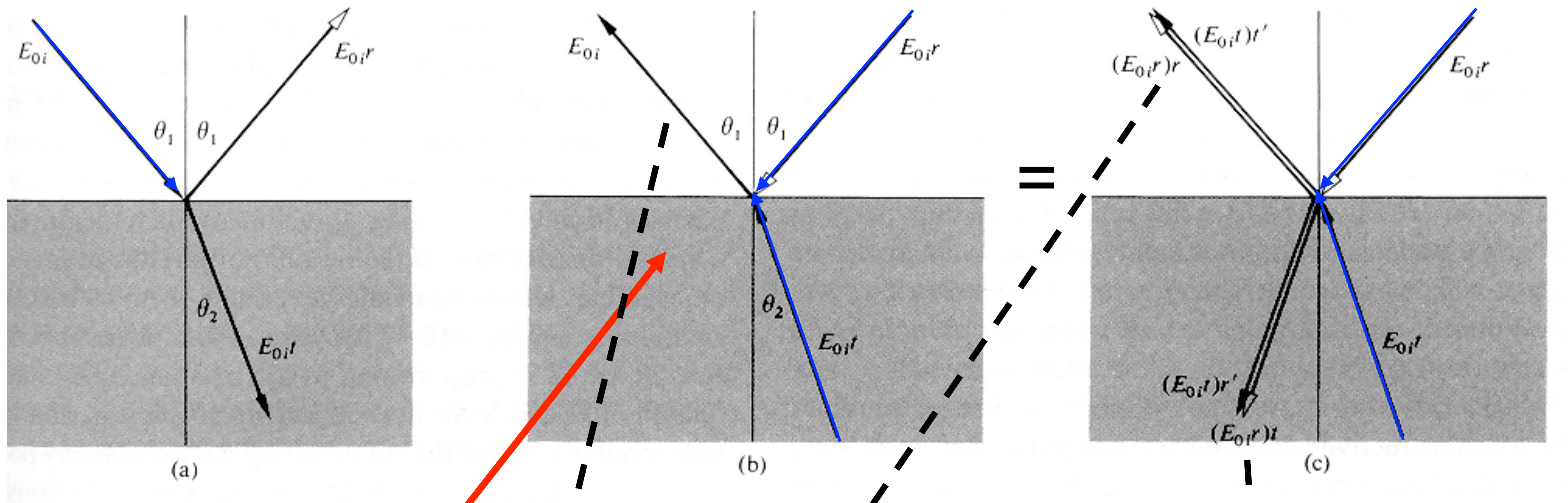
Transmitted wave:

$$E_{0t} = tt'e^{-i\delta/2}E_0\left(1+\left(r'\right)^2 e^{i\delta}+\left(\left(r'\right)^2 e^{i\delta}\right)^2+\left(\left(r'\right)^2 e^{i\delta}\right)^3+\ldots\right)$$

Reflected wave:

$$E_{0r} = rE_0 + tt'r'e^{i\delta}E_0 + tt'r'\left(\left(r'\right)^2 e^{i\delta}\right)^2 E_0 + \ldots$$

Stokes Relations for reflection and transmission



"Time reversal:"
Same amplitudes,
reversed propagation
direction

$$E_{oi} = (E_{oi} r) r + (E_{oi} t) t'$$

$$\therefore tt' = 1 - r^2$$

$$(E_{oi} t) r' + (E_{oi} r) t = 0$$

$$\therefore r' = -r$$

Notes:

- **relations apply to angles connected by Snell's Law**
- **true for any polarization, but not TIR**
- **convention for which interface experiences a sign change can vary**

Fabry-Perot transmission

Stokes' relations $r' = -r$ $\quad r'^2 = r^2$ $\quad tt' = 1 - r^2$

The transmitted wave field is:

$$E_{0t} = tt' e^{i\delta/2} E_0 \left(1 + (r')^2 e^{i\delta} + \left((r')^2 e^{i\delta}\right)^2 + \left((r')^2 e^{i\delta}\right)^3 + \ldots\right)$$

$$= tt' e^{i\delta/2} E_0 \left(1 + r^2 e^{i\delta} + \left(r^2 e^{i\delta}\right)^2 + \left(r^2 e^{i\delta}\right)^3 + \ldots\right) = tt' e^{i\delta/2} E_0 \sum_{n=0}^{\infty} \left(r^2 e^{i\delta}\right)^n$$

Where:

$$1 + x + x^2 + x^3 + \cdots = (1 - x)^{-1}$$

$$\Rightarrow \quad E_{0t} = \frac{tt' e^{i\delta/2}}{1 - r^2 e^{-i\delta}} E_0$$

Power transmittance: $T \equiv \left|\frac{E_{0t}}{E_0}\right|^2 = \left|\frac{tt' e^{i\delta/2}}{1 - r^2 e^{i\delta}}\right|^2 = \frac{(tt')^2}{(1 - r^2 e^{+i\delta})(1 - r^2 e^{-i\delta})}$

$$= \left[\frac{(tt')^2}{\{1 + r^4 - 2r^2 \cos\delta\}}\right] = \left[\frac{(1 - r^2)^2}{\{1 + r^4 - 2r^2[1 - 2\sin^2(\delta/2)]\}}\right] = \left[\frac{(1 - r^2)^2}{\{1 - 2r^2 + r^4 + 4r^2 \sin^2(\delta/2)]\}}\right]$$

Dividing numerator and denominator by $(1 - r^2)^2$

$$T = \frac{1}{1 + F \sin^2(\delta/2)}$$

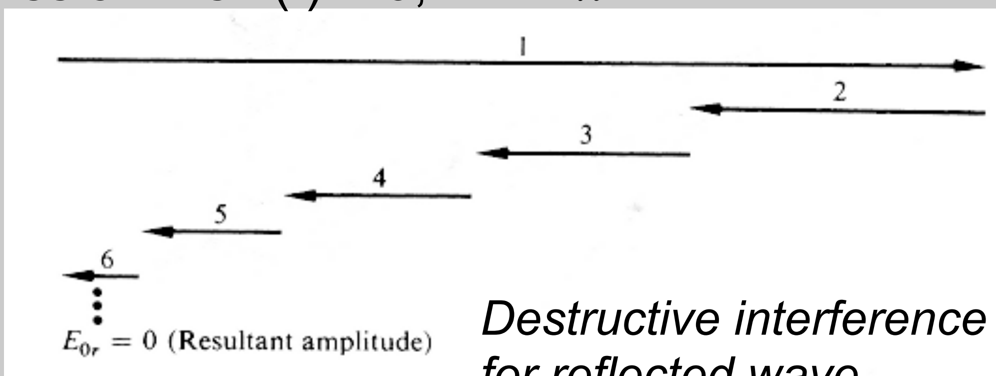
where: $F = \left[\frac{2r}{1 - r^2}\right]^2$

Multiple-beam interference: simple limits

Reflected waves

$$T = \frac{1}{1 + F\sin^2(\delta/2)}$$

Full transmission: sin( ) = 0, d = 2 π m

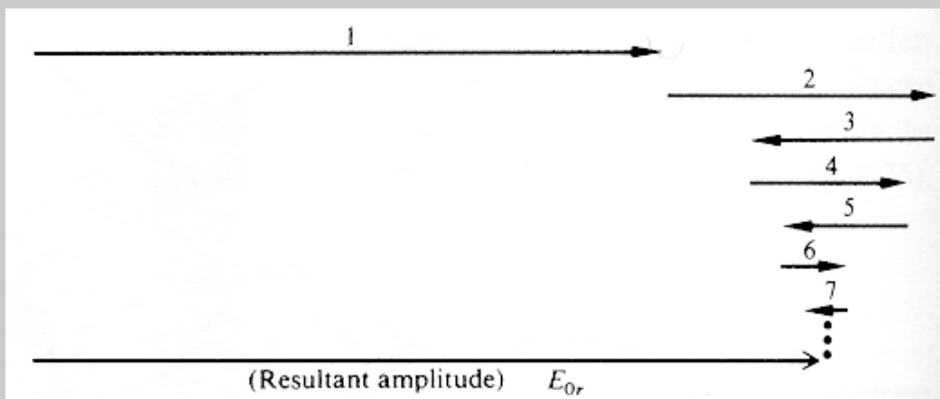


1st reflection

internal reflections

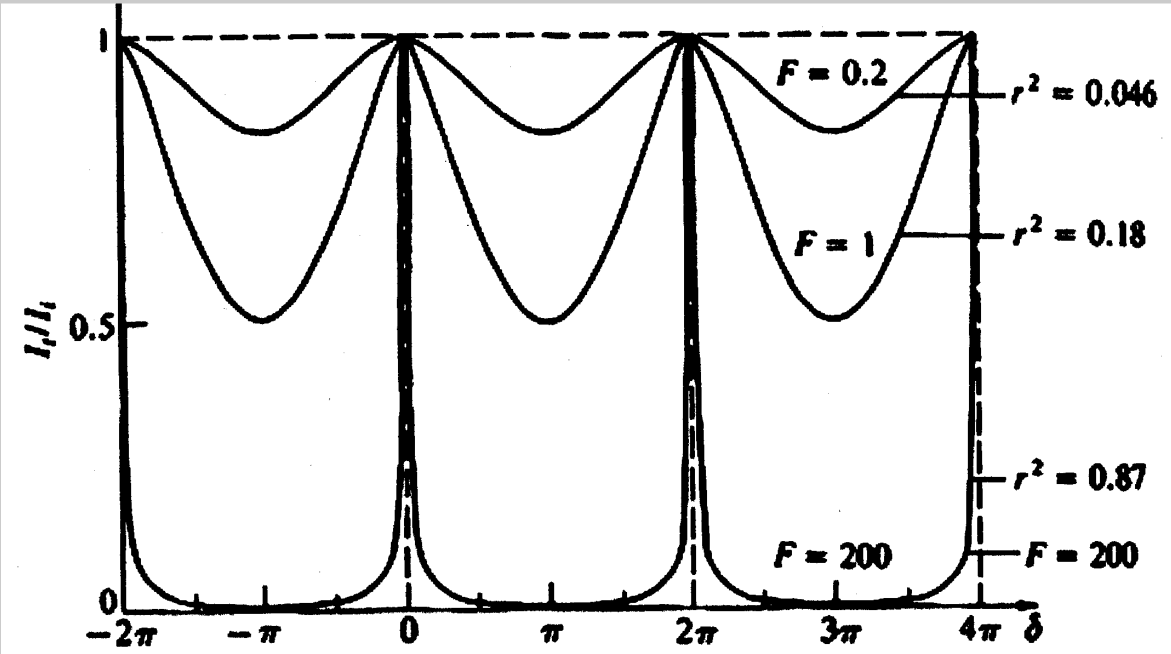
Destructive interference for reflected wave

Minimum transmission: sin( ) = 1, d = 2 π (m+1/2)



Constructive interference for reflected wave

Etalon transmittance vs. thickness, wavelength, or angle



$$T = \frac{1}{1 + F\sin^2\left(\delta/2\right)}$$

Transmission max:
sin( ) = 0, d = 2 π m

$$\delta = \frac{\omega}{c} 2nL\cos[\theta_t]$$
$$= 2\pi m$$

At normal incidence:

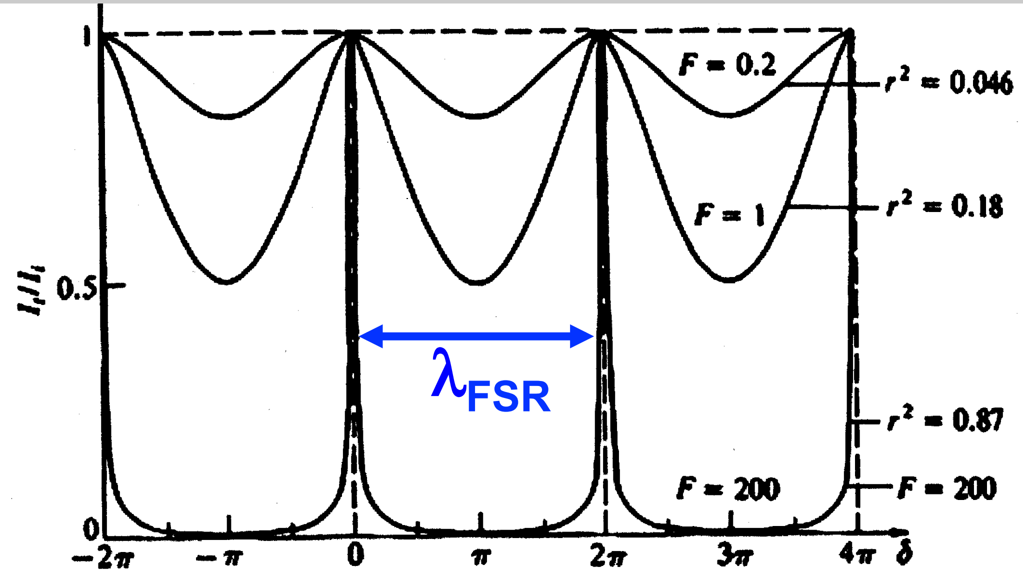
$$\lambda_m = \frac{2nL}{m} \quad \text{or} \quad nL = m\frac{\lambda_m}{2}$$

- The transmittance varies significantly with thickness or wavelength.
- We can also vary the incidence angle, which also affects δ.
- As the reflectance of each surface ($R=r^2$) approaches 1, the widths of the high-transmission regions become very narrow.

The Etalon Free Spectral Range

The Free Spectral Range is the wavelength range between transmission maxima.

λ_{FSR} = Free Spectral Range



For neighboring orders:

$$\frac{4\pi nL}{\lambda_1} - \frac{4\pi nL}{\lambda_2} = 2\pi \Rightarrow \frac{1}{\lambda_1} - \frac{1}{\lambda_2} = \frac{1}{2nL} = \frac{\lambda_2 - \lambda_1}{\lambda_1 \lambda_2}$$

$$\lambda_2 - \lambda_1 = \lambda_{FSR}$$

$$\lambda_2 \lambda_1 \approx \lambda^2$$

$$\lambda_{FSR} \approx \frac{\lambda^2}{2nL}$$

$$\frac{\lambda_{FSR}}{\lambda} = \frac{\lambda}{2nL} = \frac{\nu_{FSR}}{\nu}$$

$$\nu_{FSR} \approx \frac{c}{2nL}$$

1/(round trip time)

Etalon Linewidth

The Linewidth δ_{LW} is a transmittance peak's full-width-half-max (FWHM).

$$T = \frac{1}{1 + F\sin^2\left(\delta/2\right)}$$

- A maximum is where $\delta/2 \approx m\pi + \delta'/2$ and $\sin^2\left(\delta/2\right) \approx \delta'/2$
- Under these conditions (near resonance),

$$T = \frac{1}{1 + F\delta'^2/4}$$

- This is a Lorentzian profile, with FWHM at:

$$\frac{F}{4}\left(\frac{\delta_{LW}}{2}\right)^2 = 1 \quad \Rightarrow \quad \delta_{LW} \approx 4/\sqrt{F}$$

- This transmission linewidth corresponds to the minimum resolvable wavelength.

Etalon Finesse ≈ resolution

The Finesse, $\Im$, is the ratio of the
Free Spectral Range and the Linewidth:

δ = 2π corresponds to one FSR

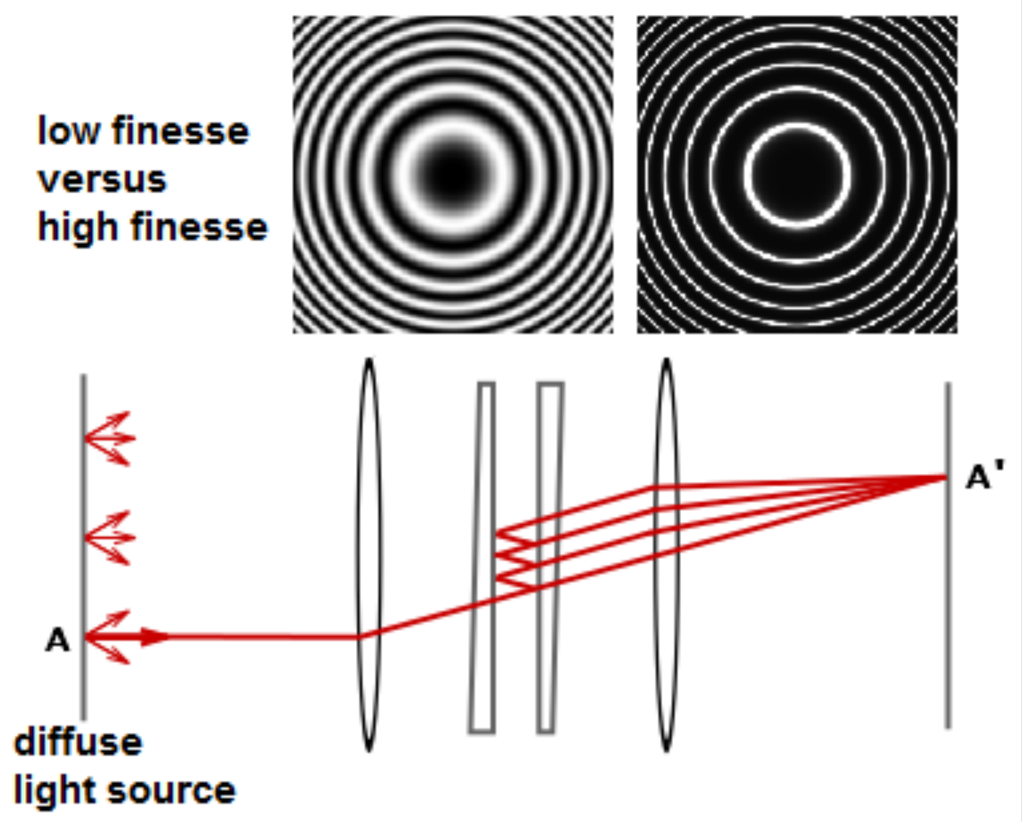
$$\Im \equiv \frac{\delta_{FSR}}{\delta_{FW}} = \frac{2\pi}{4/\sqrt{F}} = \frac{\pi\sqrt{F}}{2}$$

Using: $$F = \left[\frac{2r}{1-r^2}\right]^2$$

$$\boxed{\Im = \frac{\pi}{1-r^2}}$$ taking $r \approx 1$

The Finesse is the number of wavelengths the interferometer can resolve.

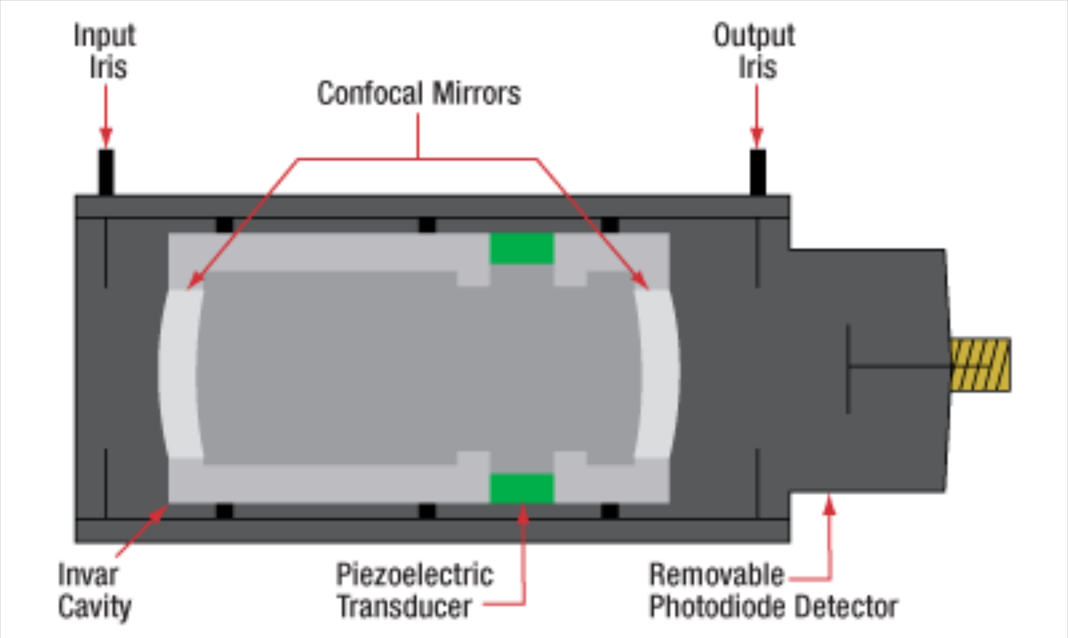
Tools: fixed plate Fabry-Perot



Tools: scanning Fabry-Perot

Resonator with piezo control over mirror separation
http://www.thorlabs.us/newgrouppage9.cfm?objectgroup_id=859

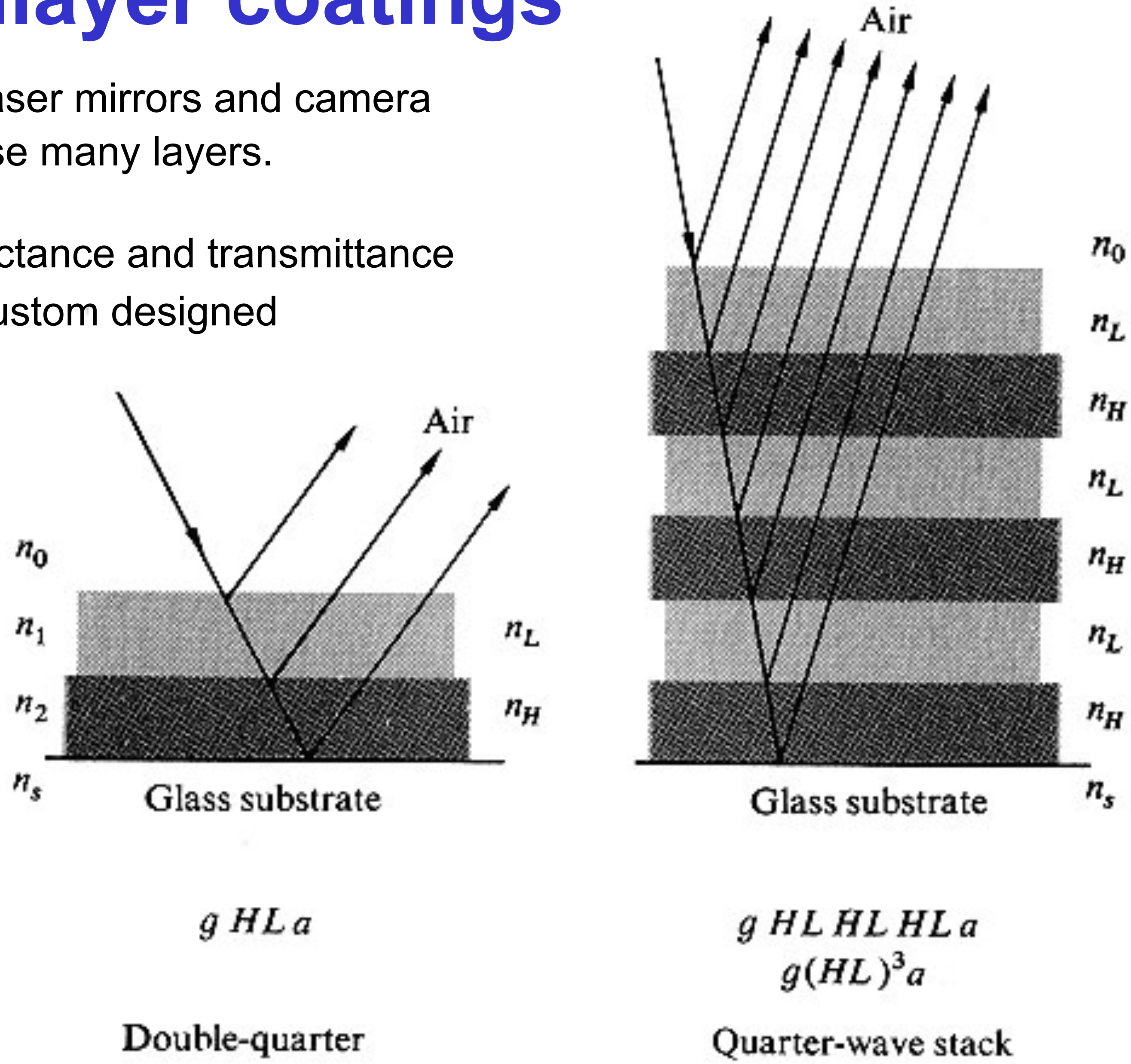
- Wavelength range: 535-820nm (ours)
- SA200 (ours)
  - FSR 1.5 GHz
  - Finesse > 200
  - Resolution 7.5MHz
- SA210
  - FSR 10 GHz
  - Finesse > 150
  - Resolution 67MHz



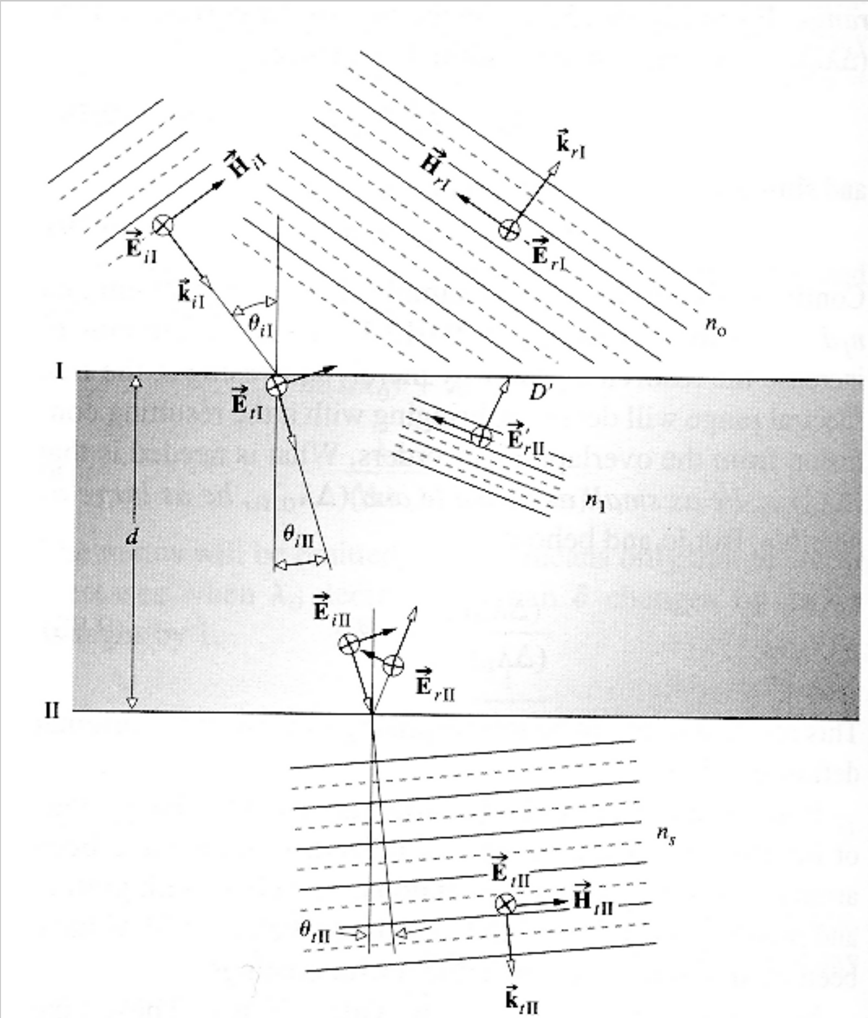
Multilayer coatings

Typical laser mirrors and camera lenses use many layers.

The reflectance and transmittance can be custom designed

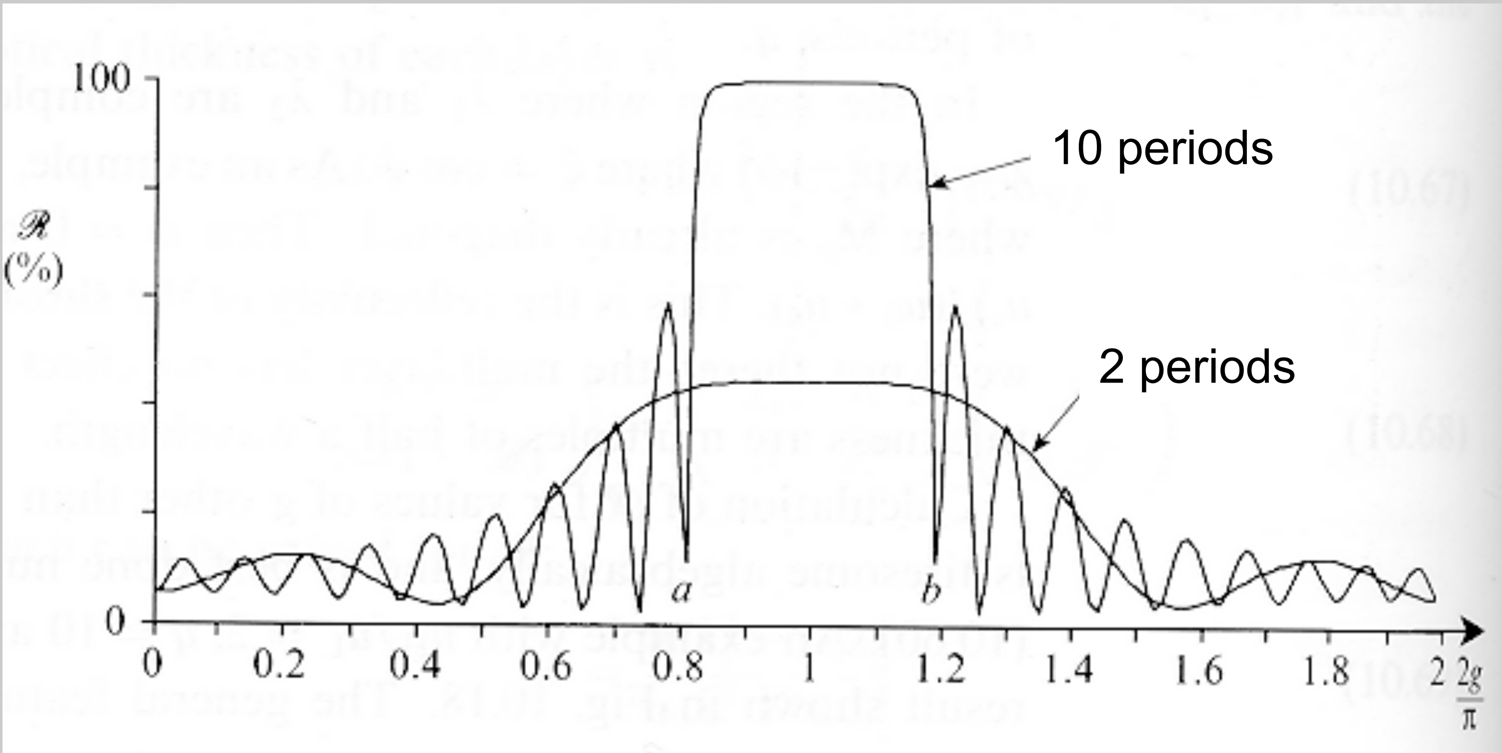


Multilayer thin-films: wave/matrix treatment



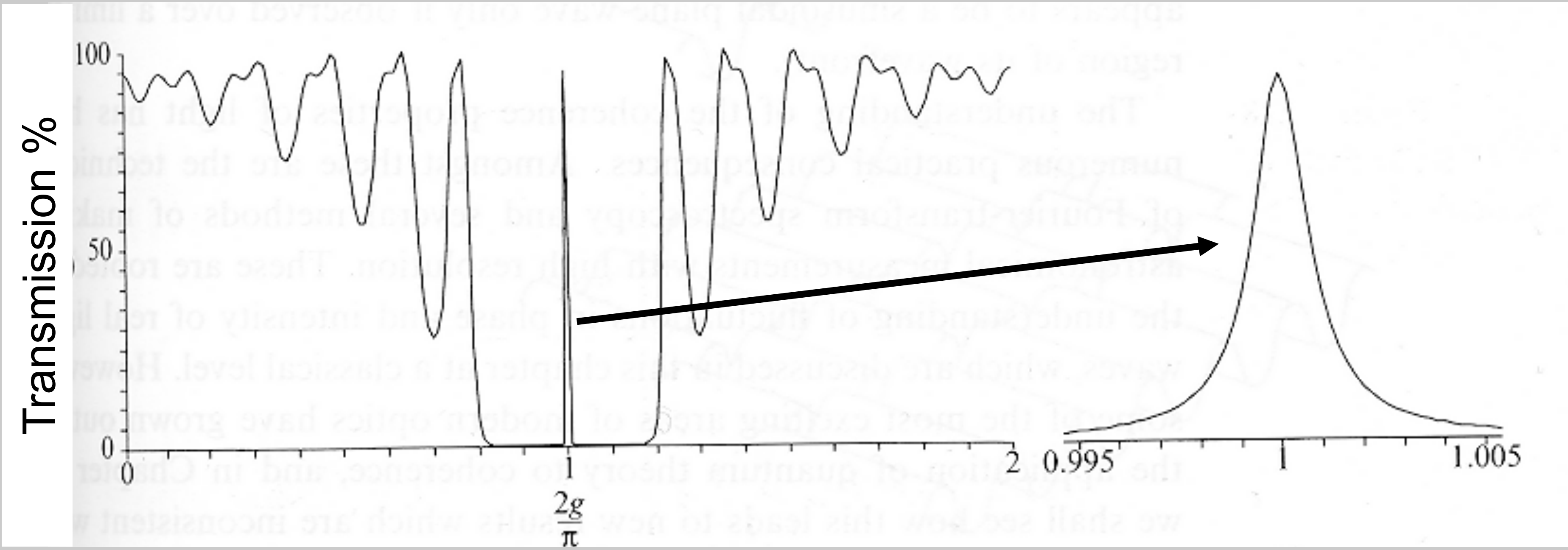
- Use boundary conditions to relate fields at the boundaries
- Phase shifts connect fields just after I to fields just before II
- Express this relation as a transfer matrix
- Multiply matrices for multiple layers

High-reflector design



Reflectivity can reach > 99.99% at a specific wavelength
> 99.5% for over 250nm
Bandwidth and reflectivity are better for “S” polarization.

Interference filter design



A thin layer is sandwiched between two high reflector coatings
-very large free spectral range, high finesse
- typically 5-10nm bandwidth, available throughout UV to IR