11-16-07 Review part 2

Note Title 11/16/2007

DFT of $\bar{f} = \{0, 1, 0\}$

$$c_k = \frac{1}{\sqrt{3}} \sum_{n=0}^{2} f_n e^{-2\pi i nk/3}$$

$$c_0 = \frac{1}{\sqrt{3}} \left[ 1 \cdot e^0 \right] = \frac{1}{\sqrt{3}}$$

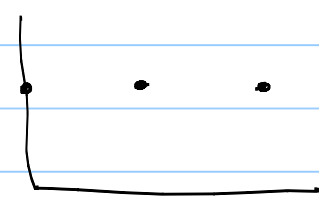
$$c_1 = \frac{1}{\sqrt{3}} \left[ 0 + 1 e^{-2\pi i/3} + 0 \right]$$

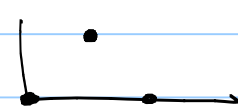
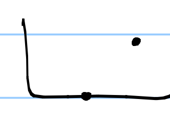
$$= \frac{1}{\sqrt{3}} e^{-i 2\pi/3}$$

$$c_2 = \frac{1}{\sqrt{3}} \left[ 0 + 1 \cdot e^{-2\pi i 2/3} \right]$$

$$= \frac{1}{\sqrt{3}} e^{-i 4\pi/3}$$

$$|c_0| = |c_1| = |c_2| = \frac{1}{\sqrt{3}}$$

periodogram = $\frac{1}{\sqrt{3}}$ 

time series =  

$$\vec{f} = \{1, 1, 1\}$$

$$c_k = \frac{1}{\sqrt{3}} \sum_{n=0}^{2} e^{-2\pi i \, nk/3}$$

$$c_0 = \frac{1}{\sqrt{3}} [1 + 1 + 1] = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$c_1 = \frac{1}{\sqrt{3}} \left[1 + e^{-2\pi i/3} + e^{-2\pi i \, 2/3}\right]$$

$$= \frac{1}{\sqrt{3}} \left[1 + e^{-2\pi i/3} + e^{-2\pi i \, 2/3}\right]$$

$$= 0$$

$$c_2 =$$

$$DFT = \{\sqrt{3}, 0, 0\}$$

Sum of N-th roots of unity

$$\sum_{k=0}^{N-1} z^k = \frac{z^N - 1}{z - 1} = 0$$

for $N=1$ $\quad \frac{z-1}{z-1} = 1$

Be able to do FT of

$\delta(t)$ $\delta(t-t_0)$

$\cos(t)$ $\cos(at)$

$\sin(t)$

$e^{-x^2}$ $e^{-ax^2}$

$e^{-ix^2}$ $e^{-ix^2}$

$e^{-|x|}$

.... ?

Basic theorems

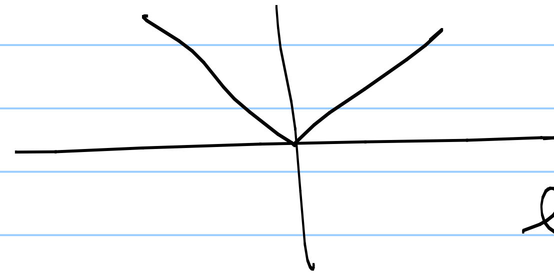
shift, Scaling

$FT(f(ax))$

$FT(f(t-t_0))$

# Fourier Series

$f(x) = |x|$
on $[-1,1]$



$\ell = 1$

$$a_0 = \int_{-1}^{1} |x|\,dx = 2\int_0^1 x\,dx = 1$$

$$a_N = \int_{-1}^{1} |x| \cos(N\pi x)\,dx$$

$$= 2\int_0^1 x \cos(N\pi x)\,dx$$

$$= 2\left[\frac{1}{N\pi}\int_0^1 x\, d(\sin(N\pi x))\right]$$

$$= \frac{2}{N\pi}\left[ x \sin N\pi x \Big|_0^1 - \int_0^1 \sin(N\pi x)\,dx \right]$$

$$\frac{2}{N\pi}\left[\ 0 - \frac{2}{\pi}\frac{1}{N} \quad \text{for } N \text{ odd}\right.$$

$$0 \quad \text{otherwise}$$

$a_0 = 1 \qquad a_1 = -\left(\frac{2}{\pi}\right)^2 \qquad a_2 = 0$

$a_3 = -\left(\frac{2}{\pi}\right)^2 \left(\frac{1}{3}\right)^2$

$a_5 = -\frac{4}{\pi^2}\ \frac{1}{25}$

$$|x| = \frac{1}{2} - \frac{4}{\pi^2}\cos(\pi x) - \frac{4}{9\pi^2}\cos(3\pi x)$$

$$- \frac{4}{25\pi^2}\cos(5\pi x) \; \cdots$$

$f(x) = x$ on $-1, 1$

$$f(x) = \frac{2}{\pi}\sin(\pi x) - \frac{1}{\pi}\sin(2\pi x)$$

$$+ \frac{2}{3\pi}\sin(3\pi x)$$

$$\cdots$$

suppose $f(x) = f(-x)$

$$F(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{-ikx}\, dx \equiv \hat{f}$$

Let $g(x) = f(x) + \hat{f}(x)$

just a var. name

$$\hat{g} = \hat{f} + \hat{\hat{f}}$$

$$\hat{\hat{f}}(q) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-iqk} \left[ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{-ikx}\, dx \right] dk$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x) \left[ \underbrace{\int_{-\infty}^{\infty} e^{-ik(q+x)}\, dk}_{2\pi\,\delta(q+x)} \right] dx$$

$$= \int_{-\infty}^{\infty} f(x)\, \delta(q+x)\, dx$$

$$= f(-q) = f(q)$$

even funct.

So $\hat{\hat{f}} = f$ and

$$\hat{g} = \hat{f} + \hat{\hat{f}} = f + \hat{f} = g$$

So $\delta$ is its own Fourier transf.

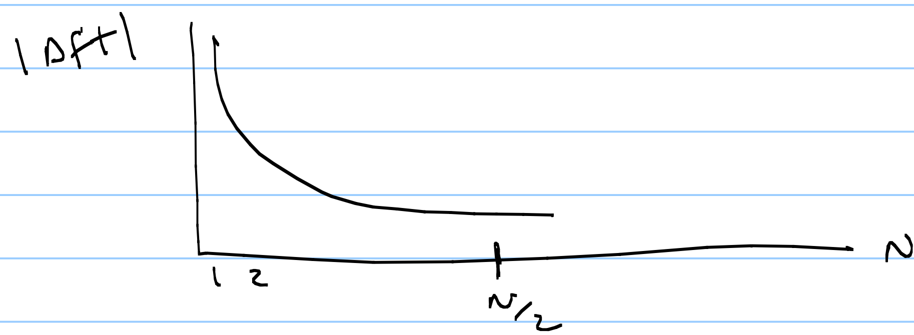
$$FT^{-1}[FT(f)] = f$$

$$FT[FT(f)] = f$$

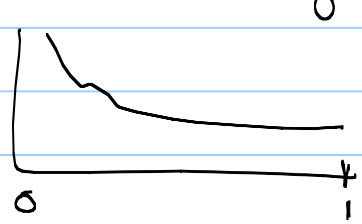
$$\int \frac{\delta(k)}{k} e^{ikx} \, dk$$

$k = 0$ $\qquad \lim_{k \to 0} \frac{e^{ikx}}{k}$

$dt = ?$




Normalize to 1



$$\delta(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega t} \, d\omega$$

$$\int_{-\infty}^{\infty} e^{i\omega[\ ]} \, d\omega = 2\pi\,\delta[\ ]$$