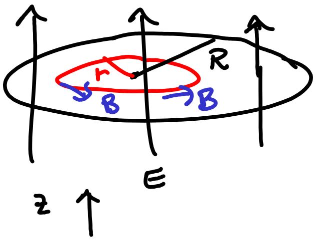
Exam 4 April 23 **Name..**

1. A uniform electric field $\vec{E}(t) = \alpha t\hat{z}$ is increasing with time and uniformly fills a circular region in the x-y plane of radius R centered at the origin. Derive an expression for the induced magnetic field for $r < R$ and specify its direction. Justify the steps in your derivation.



$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t} \qquad \vec{J} = 0 \text{ vacuum}$$

5pts

$$\int \vec{\nabla}\times\vec{B}\cdot d\vec{a} = \mu_0\epsilon_0\int\frac{\partial\vec{E}}{\partial t}\cdot d\vec{a}$$

stokes

5 pts

$$\oint\vec{B}\cdot d\vec{r} = \mu_0\epsilon_0\int_0^r \alpha 2\pi r' dr'$$

The displacement current is in the z direction and is uniform within the circle of radius R. From symmetry such a current generates a circular B. A circular path is chosen so dr is tangent to B. The dot product of B and dr is the magnitude of each times an angle theta which is zero. B is the same in magnitude for all dr so it comes outside the LHS integral.

5pts

5 pts

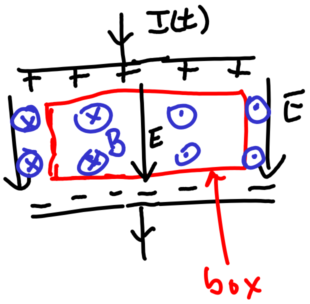
$$B 2\pi r = \mu_0\epsilon_0\alpha\frac{2\pi r^2}{2}$$

$$\vec{B} = \mu_0\epsilon_0\alpha\frac{r}{2}\,\hat{\varphi}$$

direction

5 pts

2. An imaginary box is located inside a vacuum capacitor. Write an equation in integral form for conservation of energy within the box. Begin with the work energy theorem and explain what each term means.



$$W_{net} = \Delta KE$$

$$W_{nc} + W_c = \Delta KE$$

There are no charges within the box so there is no change in KE within. There is no non-conservative force acting within the box so that term is zero.

5pts

$$0 = \frac{dW_c}{dt} = -\int u_{EM}\,d\tau - \oint\vec{S}\cdot d\vec{a}$$

$$\oint\vec{S}\cdot d\vec{a} = -\int\left(\frac{1}{2}\epsilon_0 E^2 + \frac{B^2}{2\mu_0}\right)d\tau$$

10pts

$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$ is the flow of energy per sec. It points into the box on both the left and right hand sides but not on the top or bottom sides.

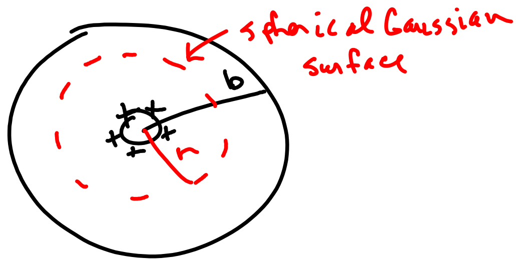
5 pts

Electromagnetic energy builds up in the box since both E and B increase. Conservation of energy then states that the EM energy flowing into the box from the left and right sides is equal to the increase in EM energy within the box.

5 pts

3. A charge Q is surronded by rubber insulation sphere out to a radius a. Assuming the rubber has suceptibility χ_e find (a) $\vec{E}$ in the rubber and (b) the bound surface charge on the outer surface of the rubber.

$\vec{\nabla}\cdot\vec{D} = \rho_{free}$ 5 pts



$\int \vec{\nabla}\cdot\vec{D}\, d\tau = \int \rho_f \, d\tau$ 5 pts

$\downarrow$ divergence th

$\oint \vec{D}\cdot d\vec{a} = Q_{f\,enclosed}$

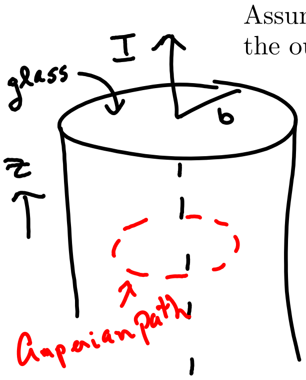
E points radially. The material is linear so P also points radially. Therefore D points radially. Choose a Gaussian surface which is a spherical shell or radius r so that the tiles on this surface have their da vector in the same direction as D. For each tile D is the same so it comes outside the integral leaving: 5 pts

$D\, 4\pi r^2 = Q_f \qquad D = \frac{Q_f}{4\pi r^2} = \epsilon E \qquad$ (a) $\vec{E} = \frac{Q_f}{4\pi\epsilon r^2} = \frac{Q}{4\pi\epsilon r^2}$ 5 pts

(b) $\rho_b = -\vec{\nabla}\cdot\vec{P} \qquad \sigma_b = \vec{P}\cdot\hat{n} \qquad \vec{P} = \epsilon_0 \chi_e \vec{E} = \epsilon_0\chi_e \frac{Q}{4\pi\epsilon r^2}\hat{r}$

$\vec{\nabla}\cdot\vec{P} = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\epsilon_0\chi_e Q}{4\pi\epsilon r^2} = 0$ cancel $\qquad \sigma_b = \vec{P}\cdot\hat{r}\Big|_{r=b} = \frac{\epsilon_0\chi_e Q}{4\pi\epsilon b^2}$ 5 pts

4. A long straight wire, carrying uniform current I_0 is surronded by glass insulation out to a radius a. Assuming the glass has suceptibility χ_m find (a) $\vec{B}$ in the glass and (b) the bound surface current on the outer surface of the glass.



$\vec{\nabla}\times\vec{H} = \vec{J}_{free}$

$\int \vec{\nabla}\times\vec{H}\cdot d\vec{a} = \int \vec{J}_f\cdot d\vec{a}$ 5 pts

$\downarrow$ stokes

$\oint \vec{H}\cdot d\vec{r} = I_f$ 5 pts

B is circular and M is linearly related to B to both point in circles around the wire. Therefore H also goes in circles. Choose an Amperian path which in a circle so that H and dr are always parallel. H is the same in magnitude for each dr so it comes out of the line integral.

$H\, 2\pi r = I_f \qquad \vec{H} = \frac{I}{2\pi r}\hat{\varphi}$

(a) $\vec{B} = \mu\vec{H} = \mu \frac{I\hat{\varphi}}{2\pi r}$ 5 pts

(b) $\vec{M} = \chi_m \vec{H} = \chi_m \frac{I}{2\pi r}\hat{\varphi}$ 5 pts $\qquad \vec{K}_b = \vec{M}\times\hat{n} = \chi_m \frac{I}{2\pi b}(-\hat{z})$ 3 pts

$\vec{I}_b = \int_0^{2\pi b} \vec{K}_b\, dr = \chi_m I(-\hat{z})$ 2 pts

5. Extra credit: Why is there or is there not a temperature effect on M for diamagnetic materials?

The temperature rotates and changes the speed of the atoms. At any instant a changing magnetic flux induces currents in the atoms in the same direction independent of the atom's orientation or speed. This generates an atomic magnetic dipole which repels it from the applied B. The orientation or speed of the atom has little effect on this induced magnetic dipole.

5pts