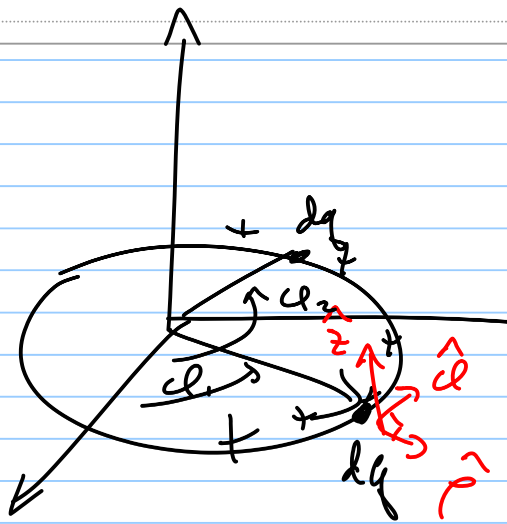



$$d\vec{E}$$

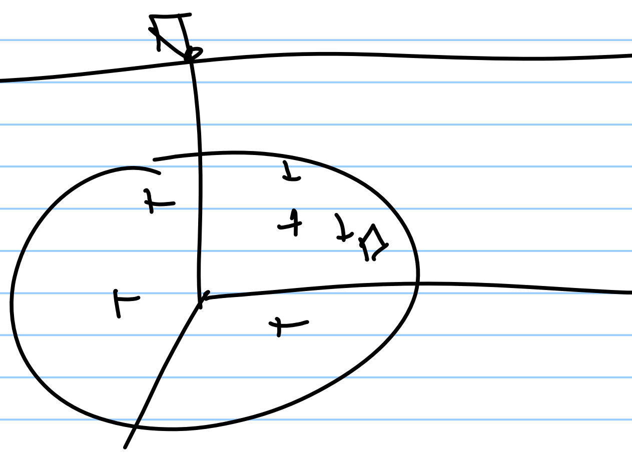
$$\vec{E}(x,y,z) = \int \frac{k\,dq}{\mathscr{r}^2}\,\hat{\mathscr{r}}$$

$$\vec{\mathscr{r}} = \vec{r} - \vec{r}\,' =$$

$$\vec{r}\,' = \underline{R}\,\hat{\rho}' + \underline{\varphi'}\,\hat{\varphi}' + \underline{z'}\,\hat{z}'$$

not constant $\quad \hat{\rho}(\varphi)$

$$\int_0^{2\pi} \hat{\rho}\quad d\varphi$$



line integrals

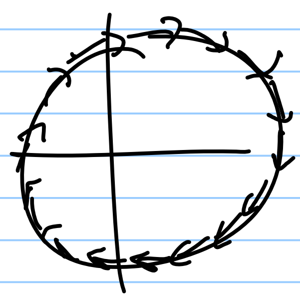
$$\int_0 \vec{E}\cdot d\vec{\ell}$$

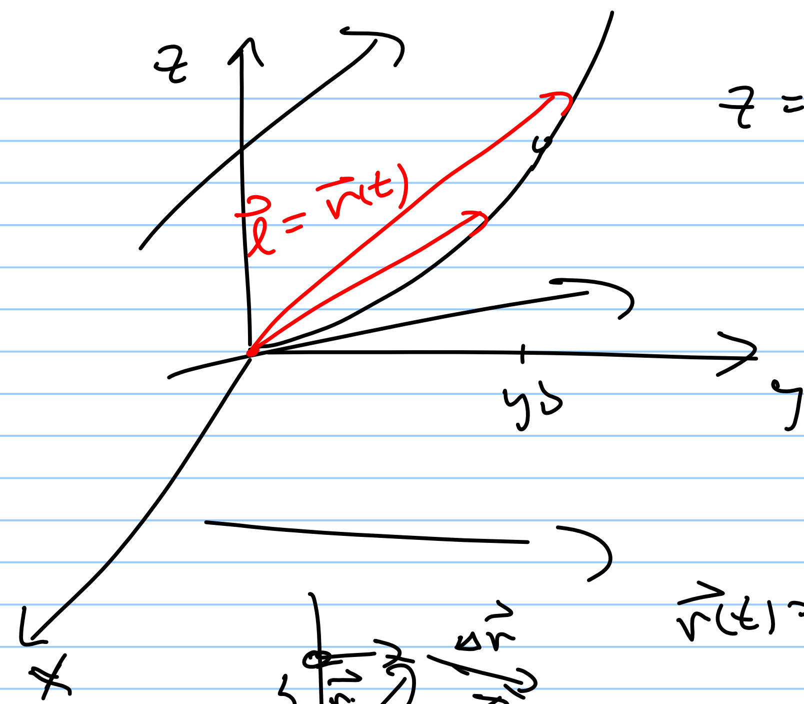
given

$$\vec{\ell} = 0\,\hat{x} + y\,\hat{y} + y^2\,\hat{z}$$

$$d\vec{\ell} = dy\,\hat{y} + 2y\,dy\,\hat{z}$$

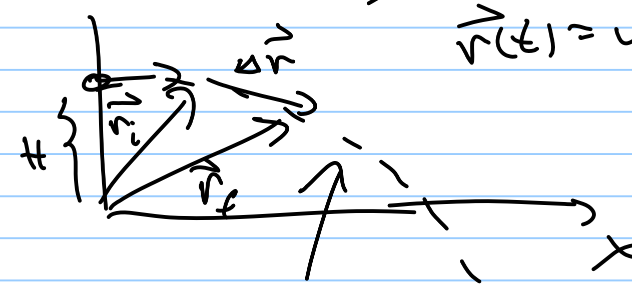
0 to $y_0$






$z = y^2$

$d\vec{r} =$



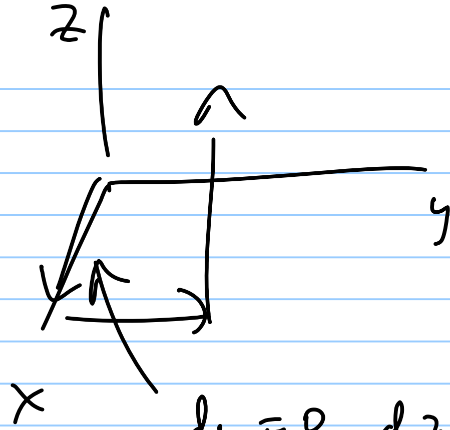

$\vec{r}(t) = v_0 t\,\hat{x} - \frac{1}{2} a t^2\,\hat{y}$

(H-

distance traveled along path

$$= \cancel{\oint d\vec{r}} = 0$$

$$= \int |d\vec{r}|$$




$d\vec{l} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$

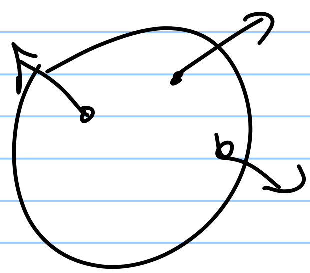
$dy = 0 \quad dz = 0 \qquad d\vec{l} = dx\,\hat{x}$

flux:

drops enclosed by surface

$$\Phi_{\text{closed surface}} = -\frac{d}{dt} D + S$$

Statics



$$\oint \vec{E}\cdot d\vec{a} = S = \frac{Q_{enc}}{\epsilon_0}$$ Gauss's law

$\vec{E}$

Divergence th.

$$\oint \vec{G}\cdot d\vec{a} = \int \vec{\nabla}\cdot\vec{G}\, d\tau$$

enclosed

$$\oint \vec{E}\cdot d\vec{a} = \int \vec{\nabla}\cdot\vec{E}\, d\tau = \frac{Q_{enc}}{\epsilon_0} = \frac{\int \rho\, d\tau}{\epsilon_0}$$

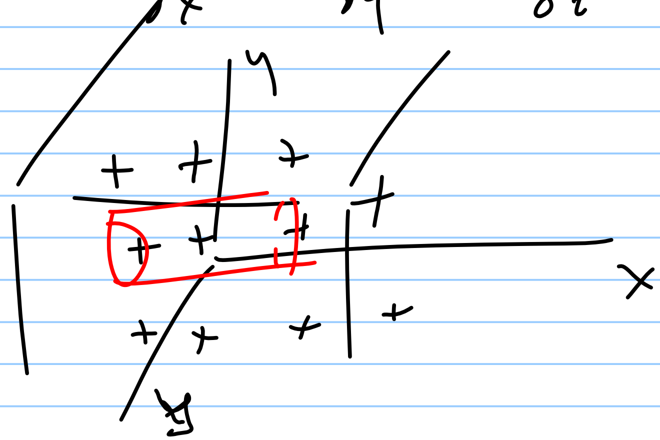
$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$ Maxwell Eqn

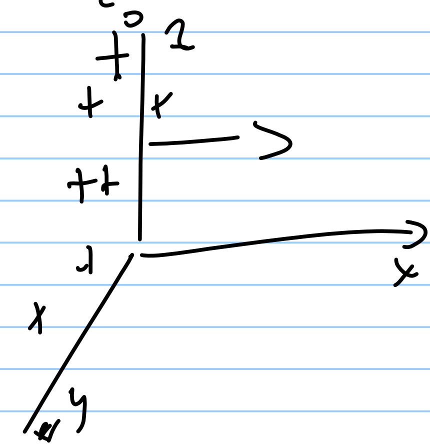
Differential form of $\oint \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$

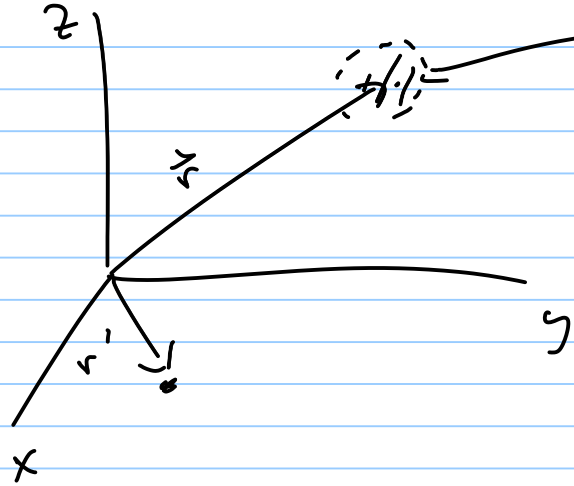
---

Ex: given $E_x = -kx$ ($k$ const) find $\rho$

$$\vec{\nabla}\cdot\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = -k = \frac{\rho}{\epsilon_0}$$






$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0} = \phi = \int \vec{\nabla}\cdot\vec{E}\, d\tau$$

$$\vec{\nabla}\cdot\vec{E} = ?$$

$$= \rho/\epsilon_0 = 0$$

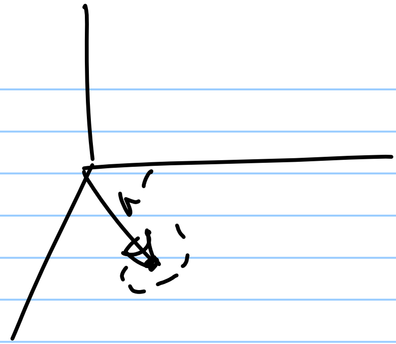
$$\hat{\mathscr{r}} = \frac{\vec{\mathscr{r}}}{|\vec{\mathscr{r}}|}$$

$$\vec{\nabla}\cdot\vec{E} = \vec{\nabla}\cdot\left(\frac{kQ}{\mathscr{r}^2}\hat{\mathscr{r}}\right) = \vec{\nabla}\cdot\frac{kQ\left[(x-x')\hat{x}+(y-y')\hat{y}+(z-z')\hat{z}\right]}{\left[(x-x')^2+(y-y')^2+(z-z')^2\right]^{3/2}}$$

$$= \frac{\partial}{\partial x}E_x + \frac{\partial}{\partial y}E_y + \frac{\partial}{\partial z}E_z = \frac{1}{[\ \ ]^{3/2}} - \frac{3}{2}\frac{2(x-x')^2}{[\ \ ]^{5/2}} + \cdots -$$

$\cancel{\partial x'}$ not x'

$$= \frac{3}{\mathscr{r}^3} - 3\frac{\mathscr{r}^2}{\mathscr{r}^5} = \phi \qquad \vec{\mathscr{r}} = \vec{r} - \vec{r}\,' = \phi$$



$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0} \neq 0$$

$$\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$$

for pt charge we need $\delta(|\vec{r}|)$

on $x, y, z$ & not $x', y', z'$

$$\rho = ?$$

charge $= \int \rho \, d\tau$

$$\frac{Q}{\epsilon_0} = \frac{1}{\epsilon_0} \int \rho \, d\tau$$

wrt $x' y' z'$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(r') \, d\tau'}{|r - r'|^2} \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{4\pi\epsilon_0} \int \rho(r') \, d\tau' \, \underbrace{\vec{\nabla} \cdot \frac{(\vec{r}-\vec{r}\,')}{|r-r'|^3}}_{4\pi \delta^3(\vec{r}-\vec{r}\,')} = \frac{Q}{\epsilon_0} = \frac{\int \rho \, d\tau}{\epsilon_0}$$

$\uparrow$ wrt $x\,y\,z$ $\qquad$ $\uparrow$

$$\frac{1}{\cancel{4\pi}\epsilon_0} \int \rho(r') \, d\tau' \, \cancel{4\pi} \, \delta^3(\vec{r}-\vec{r}\,') \qquad \underbrace{\delta^3(\vec{r}-\vec{r}\,')}$$

$$= \frac{1}{\epsilon_0} \iiint \rho'(x', y', z') \, dx' \, dy' \, dz' \, \delta(x-x') \, \delta(y-y') \, \delta(z-z')$$

$x'$ integral is $\displaystyle\int_{-\infty}^{\infty} \rho(x', z', z') \, \delta(x-x') \, dx' = \rho(x, y', z')$

The same for $y'$ & $z'$ so

$$\frac{1}{4\pi\epsilon_0} \int \rho(r') \, d\tau' \, 4\pi \, \delta(\vec{r}-\vec{r}\,') = \frac{\rho(\vec{r})}{\epsilon_0}$$

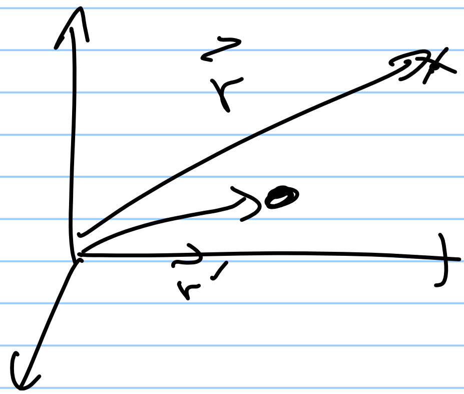
note this is not $\vec{r}-\vec{r}\,'$
nor $\vec{r}\,'$

finally

$$\vec{\nabla} \cdot \vec{E}(x,y,z) = \frac{\rho(x,y,z)}{\epsilon_0}$$

wrt $x, y, z$; $\vec{r}$

differential form of Gauss




$\delta(\ )$

$$E = \int \frac{k\,dq}{r^2}\hat{r} = \int \frac{k\,\rho\,d\tau'}{r^2}\hat{r}$$

Cons of energy:

Work energy th.

$$W_{net} = \int \underset{m\vec{a}}{\vec{F}_{net}} \cdot d\vec{l} = \Delta KE$$

$$W_{cons} + W_{non\text{-}cons} = \Delta KE$$

$\int \vec{F}_{cons} \cdot d\vec{l}$

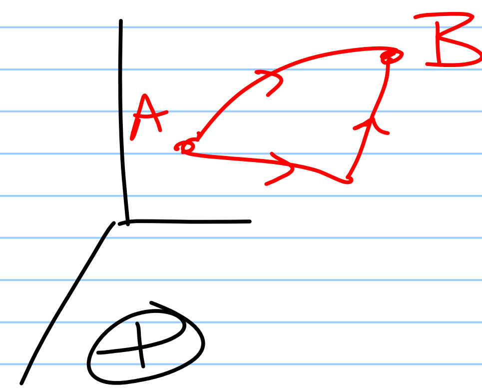
$$-\Delta PE + W_{noncon} = \Delta KE$$

$$W_{cons} = -\Delta PE$$

$$W_{non\text{-}cons} = \Delta (KE + PE)$$

Coulomb force: is it conservative?

$$W_{coul.} = \int \vec{F}_{coul} \cdot d\vec{l}$$

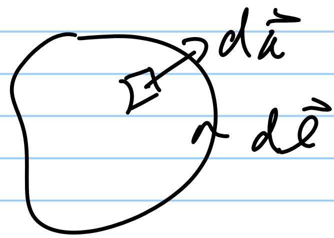



$$\oint \vec{F}_{cons} \cdot d\vec{l} = 0$$

Two major th. vector cal.

I divergence $\int \vec{\nabla} \cdot \vec{G}\, d\tau = \oint \vec{G} \cdot d\vec{a}$

II Stokes $\int \vec{\nabla} \times \vec{G} \cdot d\vec{a} = \oint \vec{G} \cdot d\vec{l}$



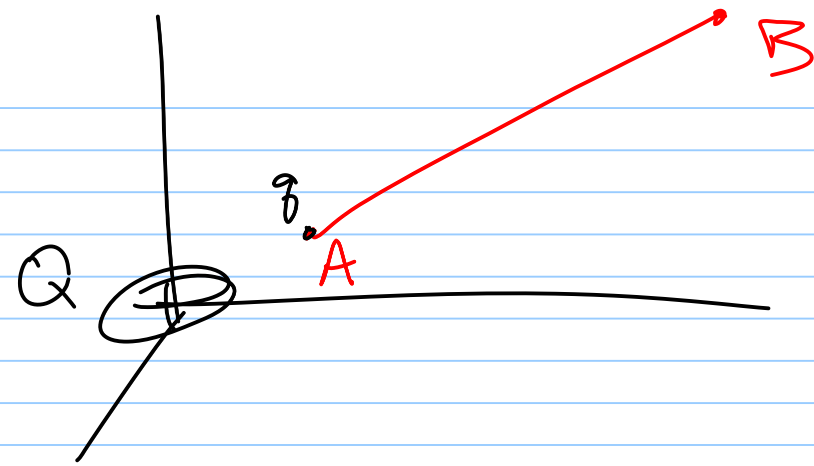

flux of $\vec\nabla \times \vec{G}$ thru $da$

$$\vec\nabla \times \vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = 0$$

$$\vec{E} = \frac{1}{4\pi\epsilon} \frac{Q\left\{(x-x')\hat{x} + (y-y')\hat{y} + (z-z')\hat{z}\right\}}{r^3}$$

$$W_{coul} = -\Delta PE$$

Ex:




$$W = \int \vec{F}_{coul} \cdot d\vec{l} = \int_A^B q\frac{kQ}{r^2}\hat{r} \cdot dr\,\hat{r}$$

$$= qkQ\int_A^B \frac{dr}{r^2} = qkQ\left(-\frac{1}{r}\right)\Bigg|_A^B = qkQ\left(\frac{-1}{r_B} + \frac{1}{r_A}\right)$$