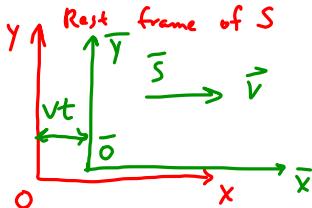


Reading
up to 12.3 (pg. 522)

Lorentz Transformation

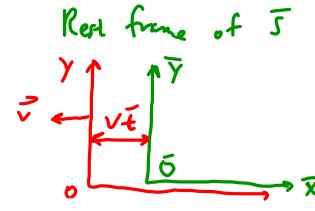


$$\bar{x} = \gamma(x - vt) \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\bar{y} = y$$

$$\bar{z} = z$$

$$\bar{t} = \gamma(t - \frac{vx}{c^2})$$



$$x = \gamma(\bar{x} + vt)$$

$$y = \bar{y}$$

$$z = \bar{z}$$

$$t = \gamma(\bar{t} + \frac{v\bar{x}}{c^2})$$

① Synchronization

at $t=0$, an observer in S

looks at all the clocks in \bar{S}

$$\bar{t} = -\frac{vx}{c^2}\gamma$$

$x > 0$ then \bar{S} 's clocks are "behind"

$x < 0$ then \bar{S} 's " " " " ahead"

② Length Contraction

Stick of length $\Delta\bar{x} = \bar{x}_r - \bar{x}_l$ in \bar{S} (at rest in \bar{S})

In S's frame: measure the length at a specific time

$$\Delta x = \gamma(x_r - vt) \quad \Delta t = 0$$

$$\Delta x = \gamma \Delta x$$

$\Delta x = \frac{\Delta \bar{x}}{\gamma}$ rest length In a moving frame, objects are shorter by a factor of γ .

③ Time Dilation

Observer in S looks at one specific clock in \bar{S} .

$$\Delta t = \gamma (\Delta \bar{t} + v \frac{\Delta \bar{x}}{c^2}) \quad \Delta \bar{x} = 0$$

$$= \gamma \Delta \bar{t} \quad \text{Time interval gets longer when measured in a moving frame.}$$

④ Simultaneity

In frame S

Event A: $x_A = 0, t_A = 0$

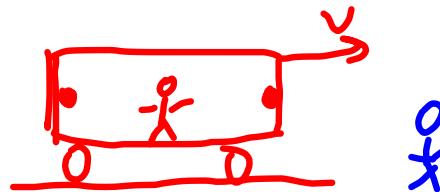
Event B: $x_B = b, t_B = 0$

Simultaneous in S

In frame \bar{S}

A: $\bar{x}_A = 0, \bar{t}_A = 0$

B: $\bar{x}_B = \gamma b, \bar{t}_B = -\frac{\gamma v b}{c^2}$



Example

(a) Show that for an ultra-relativistic particle $\beta \rightarrow 1$ ($\rho = \frac{v}{c}$)

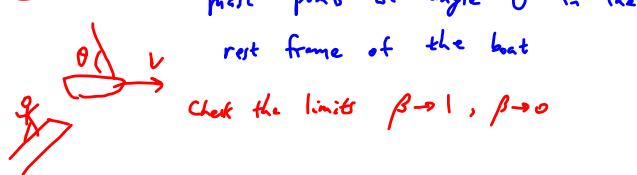
$$\beta = 1 - \frac{1}{2\gamma^2}$$

(b) If you want a particle to last 10 times its normal lifetime in a lab, how fast does it have to go in your frame (β)?

$$\sigma t = \gamma \sigma \bar{t} \quad \gamma = 10$$

$$\beta = 1 - \frac{1}{2\gamma^2} = 1 - \frac{1}{200} = \frac{199}{200}$$

12.10



rest points at angle θ in the
rest frame of the boat

check the limits $\beta \rightarrow 1$, $\rho \rightarrow 0$

Velocity addition rule

In frame S , particle moves w/ velocity

$$\frac{dx}{dt} = u$$

What is \bar{u} (velocity in \bar{S})

$$\begin{aligned}\frac{d\bar{x}}{dt} &= \frac{\gamma(dx - vt)}{\gamma(dt - \frac{v}{c^2} dx)} \quad \text{divide by } dt \\ &= \frac{u - v}{1 - \frac{vu}{c^2}}\end{aligned}$$

12.15

A hand-drawn diagram showing a bullet train moving to the right with a velocity vector labeled v . A bullet is shown being fired from the front of the train with a velocity vector labeled u . The ground is represented by a horizontal line.

		Cops	rollers	
		$\bullet \bullet$	$\bullet \bullet$	
speed of \rightarrow		Ground	Cops	rollers
relative to \downarrow				bullet
Ground		0	$\frac{1}{2}c$	$\frac{3}{4}c$
Cops		$-\frac{1}{2}c$	0	$\frac{2}{3}c$
rollers		$-\frac{3}{4}c$	$-\frac{2}{3}c$	0
bullet				$-\frac{1}{12}c$
$U_{\text{bullet, ground frame}}$				

$$\bar{u} = \frac{u - v}{1 - \frac{vu}{c^2}}$$

velocity in the "new" frame
 u = velocity of the object in the "old" frame

v = velocity of the new frame
relative to the old frame