MATH 225 – Worksheet #4 Name:_____

Sec_____

To get full credit, you must show all of your work.

1. Solve the initial value problems explicitly:
$\frac{dy}{dt} - \tan(t)y = \cos(t), y(0) = 8, 0 \le t < \frac{\pi}{2}.$

2. Consider the differential equation, $ty' = t^2y - 2t^2$. Which two methods of solution can you use to solve this problem? Solve using both methods to check your answers.

3. A tank with a capacity of 400 gallons initially contains 100 gallons of water in which 10 pounds of salt is dissolved. Water containing 2 lb of salt per gallon is flowing into the tank at the rate of 4 gallons per minute. The well-stirred mixture leaves the tank at the rate of 2 gallons per minute.

   a. Write down the initial value problem that models the dynamics of the tank.

   b. Solve the initial value problem from (a).

   c. When will the tank start overflowing?

   d. How much salt is in the tank at the point of overflowing? Simplify your answer.

4. Consider the predator-prey model defined by

$$\frac{dx}{dt} = -0.1x + 0.02xy$$

$$\frac{dy}{dt} = 0.2y - 0.025xy$$

   a. Which variable represents the prey and which represents the predator? Why?

   b. What are the equilibrium solutions of the system?

5. Consider the differential equation $y'' + y' + 2y = 0$.

   a. Show that $y(t) = e^{-t/2}\cos\left(\frac{t\sqrt{7}}{2}\right)$ is a solution.

   b. Let $v = \frac{dy}{dt}$ and convert the second-order differential equation into a first-order system.