Hmwk 7 Solns

1.) This will be covered in class on Mar 26 thru Mar 27.

2.) Use $\vec{\nabla}\cdot\vec{D} = \rho_{free} \longrightarrow \oint \vec{D}\cdot d\vec{a} = Q_{free}$

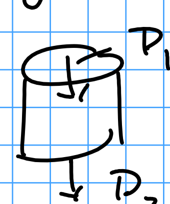
(A) Assume a linear material $\vec{P} = \epsilon_0 \chi_e \vec{E}$

(B) Determine the direction of $\vec{D} = \epsilon_0\vec{E} + \vec{P}$

(C) Useful only if the problem has enough symmetry that the direction of $\vec{D} = \epsilon_0\vec{E} + \vec{P}$ can be determined.

(d) Choose a Gaussian surface such that tiles on this surface either have $\vec{D} \perp d\vec{a}$ or $\vec{D} \parallel d\vec{a}$.
Enclose some Q_{free} with this Gaussian surface. If you don't then $\oint \vec{D}\cdot d\vec{a} = 0$ & you most likely will end up with

$$\underset{\uparrow\ D \text{ at one surface}}{D_1} A - D_2 \overset{\text{area}}{A} = 0 \quad \text{or} \quad D_1 = D_2$$



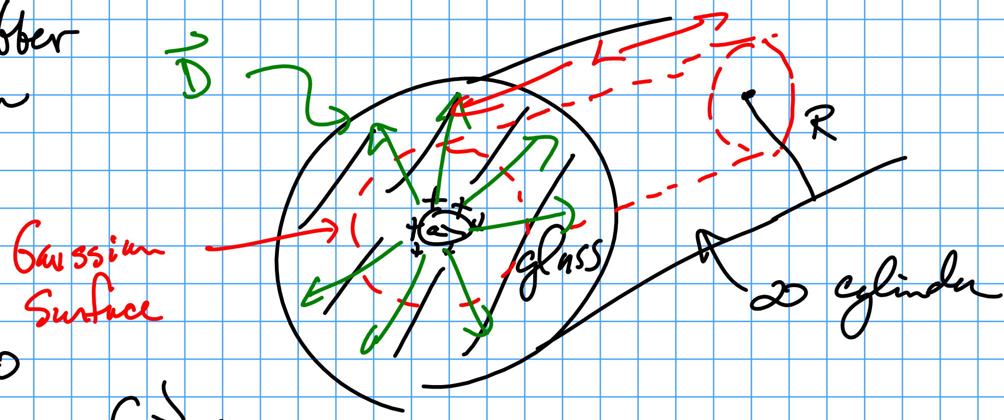
From this you will not learn much

(e) Apply $\oint \vec{D}\cdot d\vec{a} = Q^{enclosed}_{free}$

solve for $D = \epsilon E$ where $\epsilon = \epsilon_0(1+\chi_e)$

in terms of $Q_{free}^{enclosed}$ & then find $E = \frac{D}{\epsilon}$

Ex: ∞ cylinder of rubber around a wire with $\sigma_f \left(\frac{C}{m^2}\right)$



$$\oint \vec{D}\cdot d\vec{a} = \underbrace{\cancel{\int \vec{D}\cdot d\vec{a}}}_{\substack{\text{cap} \\ \vec{D}\perp d\vec{a}}}^{\;0} + \underbrace{\int \vec{D}\cdot d\vec{a}}_{\text{body } \parallel}$$

$$D\, 2\pi r L = \sigma_f 2\pi a L$$

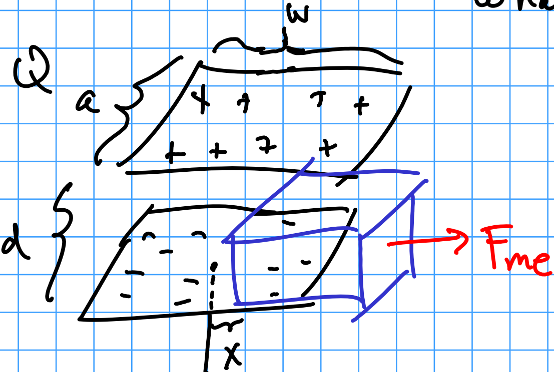
$D = \sigma_f \frac{a}{r} = \epsilon E$ $\qquad E = \sigma_f \frac{a}{r} \frac{1}{\epsilon_0(1+\chi_e)} \quad r < R$

$E = \frac{\sigma_f}{\epsilon_0}\frac{a}{r} \quad r > R$

3.) $W_{non\text{-}cons\ force} = W_{me} = \Delta(KE + PE)$ move glass at const v so $\Delta KE = 0$

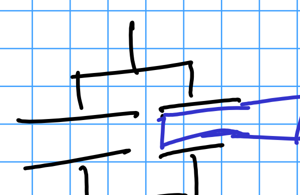
$PE = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$ Choose one or the other based on what is constant in the problem

Const Q



$$\Delta PE \approx \frac{dPE}{dC}\frac{dC}{dx}\Delta x$$
$$\frac{dPE}{dC} = -\frac{1}{2}\frac{Q^2}{C^2}$$

What is $C(x)$? Think of this as 2 caps in parallel or



$$C(x) = \epsilon_0 \frac{a\left(\frac{w}{2}+x\right)}{d} + \epsilon \frac{a\left(\frac{w}{2}-x\right)}{d}$$

$$C(x) = \epsilon_0 \frac{a}{d}\frac{w}{2} + \frac{\epsilon_0 a}{d}x + \epsilon_0(1+\chi_e)\frac{a\frac{w}{2}}{d} - \frac{\epsilon_0(1+\chi_e)ax}{d} = \epsilon_0\chi_e\frac{a}{d}\left(\frac{w}{2}-x\right) + \epsilon_0\frac{a}{d}w$$

subtract

add

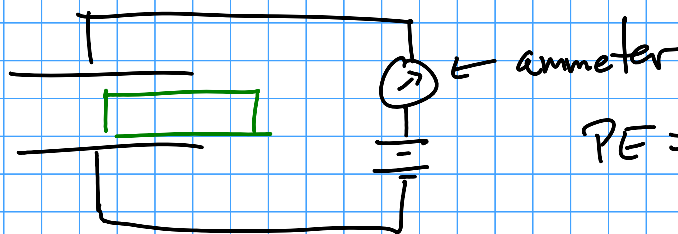
$$\frac{dC}{dx} = -\epsilon_0\chi_e\frac{a}{d}$$

So $\Delta PE \simeq \frac{dPE}{dC}\frac{dC}{dx}\Delta x = \frac{1}{2}\frac{Q^2}{C^2}\epsilon_0\chi_e\frac{a}{d}\Delta x = F_{me}$

$$\left(-\frac{1}{2}\frac{Q^2}{C^2}\right)\left(-\epsilon_0\chi_e\frac{a}{d}\right)\Delta x$$

$C(x)$ given above

For const. Voltage



$PE = \frac{1}{2}CV^2$

Note however that as glass is removed the battery has to take back charge so work is also done on battery $\Delta Q V$

$$W_{non\ cons} = W_{me} + W_{battery} = \Delta PE$$

$$F_{me}dx + dQ\,V = d\,PE_{cap}$$

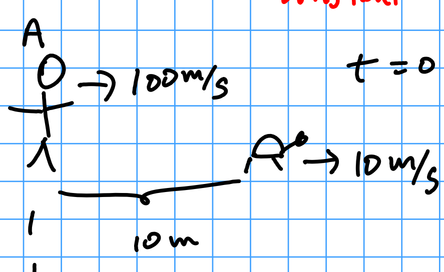
$$F_{me} = \frac{d\,PE_{cap}}{dx} - V\frac{dQ}{dx}$$

negative since cap decreases as glass is withdrawn

Note $Q = CV$ so $\frac{dQ}{dx} = \frac{dC}{dx}V$ use $C(x)$ above & see text on energy in dielectrics

constant

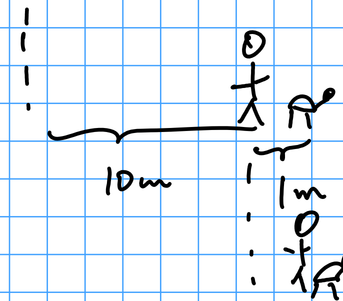
4.)



$t = 0$

$vt = \text{dist}$

$100t = 10$



$t = \frac{1}{10}$

$100\frac{m}{s}\, t = 1m$

$t = \frac{1}{100}$

next time is $t = \frac{1}{1000}$

time for Achilles to reach turtle $= \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \cdots$

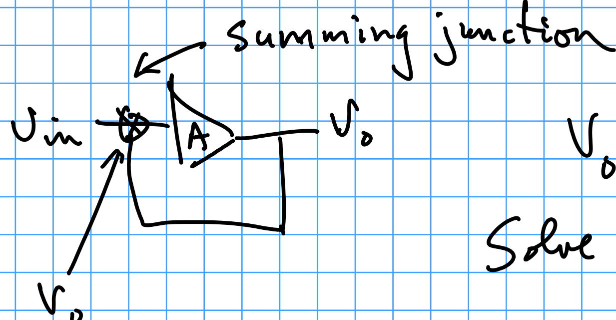
which is an ∞ series. Zeno thinks infinite terms $\Rightarrow$ infinite time

However series $= .111111\ldots$ which is a finite time.

5.)

$V_o = A V_{in}$

Now introduce feedback



Positive feedback (add)

$V_o = A V_{in} + A V_o$

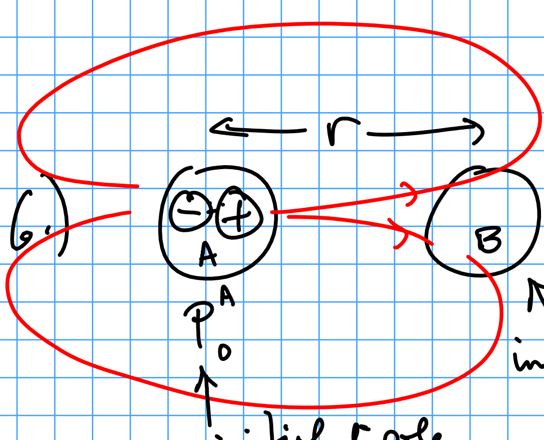
Solve for $V_o = \frac{A V_{in}}{1 - A}$ which $\to \infty$ as $A \to 1$

For small A V_o is finite even with positive feedback.

Taylor series expansion of $\frac{1}{1-A} \simeq 1 + A + A^2 + A^3 + \cdots$

perturbation expansion for small A

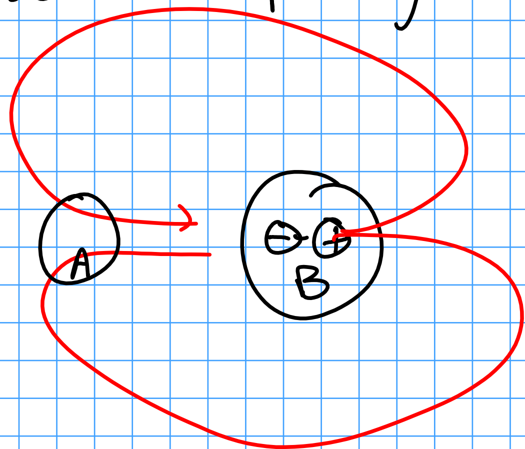
6.)



E_{dipole}

induced dipole $p_0^B = \alpha E_0^A$ where $E_0^A = \frac{2 p_0^A}{4\pi\epsilon_0 r^3}$

Now B has dipole p_0^B and this generates E_0^B which acts on A to increase A's dipole beyond p_0^A by an amount $p_1^A = \alpha E_0^B$



where $E_0^B = \dfrac{2p_0^B}{4\pi\epsilon_0 r^3}$

Now A has extra dipole p_1^A and this generates E_1^A which acts on B to increase B's dipole beyond p_0^B an amount $p_1^B = \alpha E_1^A$

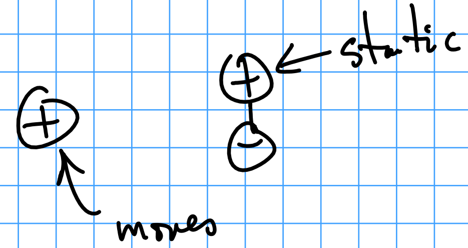
This process continues with the final dipole moment for A

$$p^A = p_0^A + p_1^A + p_2^A + \cdots$$

$$p^B = p_0^B + p_1^B + p_2^B + \cdots$$

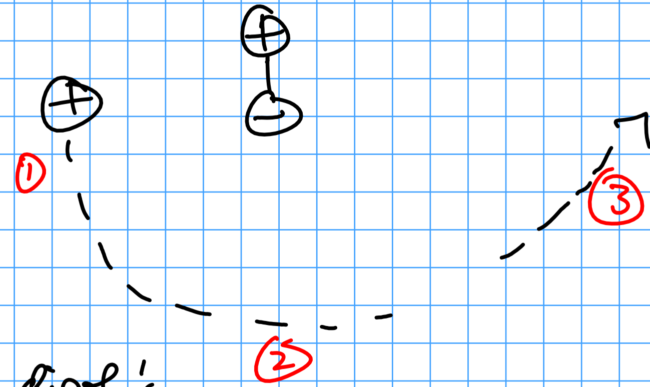
Like Zeno's paradox and the amplifier with positive feedback this series can sum to a finite number even with the positive feed back of one dipole on the other.

7.) See lecture notes on how Electric field hockey is coded. Need to add torque ODE's or use Newton's 3rd law with Electric field hockey. That is

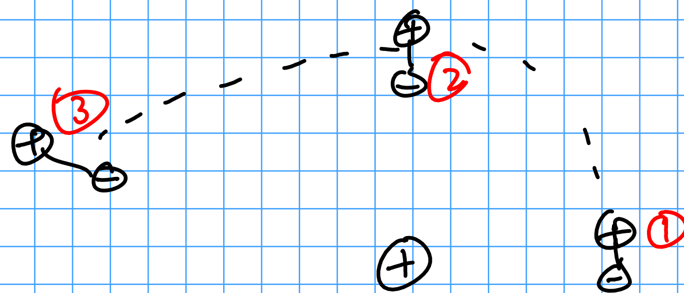


Go to frame of moving point charge to see how dipole moves

For example



in frame of pt charge dipole's position is



9.) See lecture notes March 22