Maxwell's Eqns

Reading: G 7.3

Tomorrow: G 8.1

① $\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0$ ← 1 eqn

electric field

charge density

② $\vec{\nabla} \cdot \vec{B} = 0$ ← 1 eqn

magnetic field

③ $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ ← 3 eqns

④ $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$ ← 3 eqns

Used for finding $\vec{E}, \vec{B}$ from $\rho, \vec{J}$.

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

$$q = \int \rho \, dV \quad [C]$$

Coulomb

$$\vec{f} = \rho\vec{E} + \vec{J} \times \vec{B}$$

force per volume

integral of this is the first eqn.

1. $$\int \vec{\nabla}\cdot\vec{E}\,dV = \int \frac{\rho}{\epsilon_0}\,dV$$

$$\oint \vec{E}\cdot d\vec{a} = \int \frac{\rho}{\epsilon_0}\,dV$$

$$\oint \vec{E}\cdot d\vec{a} = \frac{1}{\epsilon_0} Q_{enc}$$

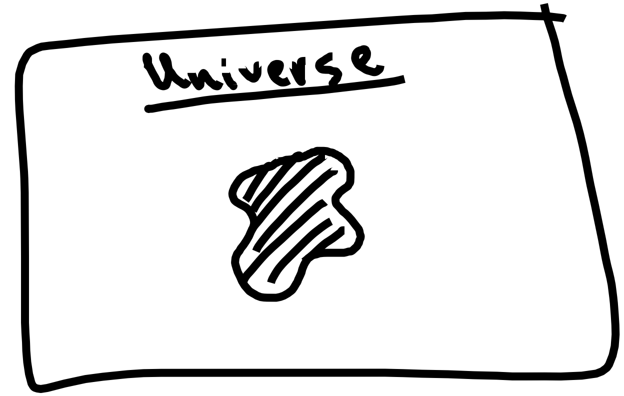
2. $$\int \vec{\nabla}\cdot\vec{B}\,dV = \int 0\,dV$$

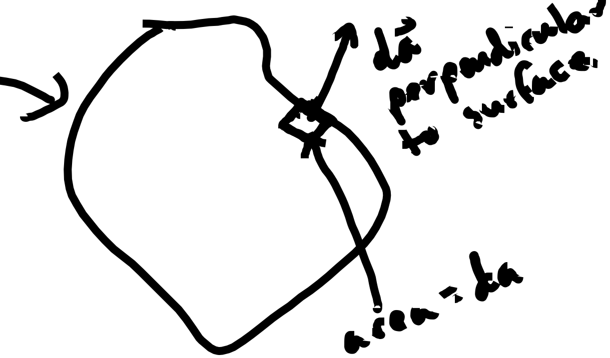
$$\oint \vec{B}\cdot d\vec{a} = 0$$

3. $$\int (\vec{\nabla}\times\vec{E})\cdot d\vec{a} = \int -\frac{\partial \vec{B}}{\partial t}\cdot d\vec{a}$$

$$\oint \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{a} = -\frac{\partial \Phi_B}{\partial t}$$

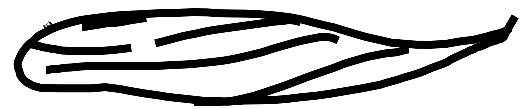
4. $$\oint \vec{B}\cdot d\vec{l} = \mu_0\int \vec{J}\cdot d\vec{a} + \mu_0\epsilon_0\int \frac{\partial \vec{E}}{\partial t}\cdot d\vec{a}$$





Newton's 2nd Law

$$\sum \vec{F} = \frac{d\vec{p}}{dt}$$



$\rho = \rho_f + \rho_b$

$\uparrow$ free $\quad \uparrow$ bound

Polarization: $-\vec{\nabla} \cdot \vec{P} = \rho_b$

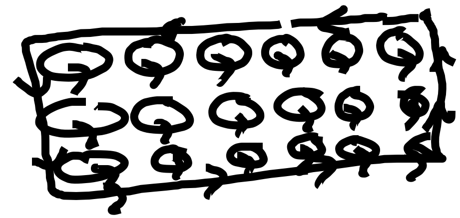
Electric Displacement:

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$\vec{\nabla} \cdot \vec{D} = \vec{\nabla} \cdot (\epsilon_0 \vec{E} + \vec{P})$$

$$= \rho - \rho_b = \rho_f$$

$$\vec{J} = \vec{J}_b + \vec{J}_f + \vec{J}_p$$



$$\vec{\nabla} \cdot \vec{J}_p = -\frac{\partial \rho_b}{\partial t}$$

$\uparrow$ continuity eqn.

A statement of conservation of ρ_b.

Define Magnetic Displacement:

$$\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M} \quad ; \quad \vec{\nabla} \times \vec{M} = \vec{J}_b$$

Derive Maxwell's eqns in terms of $\vec{D}$, $\vec{H}$, ρ_f, $\vec{J}_f$.

$\vec{E} = \frac{1}{\epsilon_0}\vec{D} - \frac{1}{\epsilon_0}\vec{P}$ $\qquad$ $\vec{B} = \mu_0\vec{H} + \mu_0\vec{M}$

① $\vec{\nabla}\cdot\left(\frac{1}{\epsilon_0}\vec{D} - \frac{1}{\epsilon_0}\vec{P}\right) = \frac{1}{\epsilon_0}\vec{\nabla}\cdot\vec{D} + \frac{1}{\epsilon_0}\rho_b = \frac{1}{\epsilon_0}\rho$

$\Rightarrow \boxed{\vec{\nabla}\cdot\vec{D} = \rho_f}$

② $\vec{\nabla}\cdot\left(\mu_0\vec{H} + \mu_0\vec{M}\right) = \phi \rightarrow \vec{\nabla}\cdot\vec{H} \stackrel{?}{=} \phi$, Not really $\quad \vec{\nabla}\cdot\vec{B} = \phi$

③ $\vec{\nabla}\times\left(\frac{1}{\epsilon_0}\vec{D} - \frac{1}{\epsilon_0}\vec{P}\right) = -\frac{\partial}{\partial t}\left(\mu_0\vec{H} + \mu_0\vec{M}\right)$

$\frac{1}{\epsilon_0}\vec{\nabla}\times\vec{D} - \frac{1}{\epsilon_0}\vec{\nabla}\times\vec{P} = -\mu_0\frac{\partial}{\partial t}\vec{H} - \mu_0\frac{\partial\vec{M}}{\partial t}$

$\frac{1}{\epsilon_0}\vec{\nabla}\times\vec{D} = -\mu_0\frac{\partial\vec{H}}{\partial t} + \frac{1}{\epsilon_0}\vec{\nabla}\times\vec{P} - \mu_0\frac{\partial\vec{M}}{\partial t}$

$\vec{\nabla}\times\vec{D} = -\frac{1}{c^2}\frac{\partial\vec{H}}{\partial t} + \vec{\nabla}\times\vec{P} - \frac{1}{c^2}\frac{\partial\vec{M}}{\partial t}$

$c = \sqrt{\frac{1}{\mu_0\epsilon_0}}$ = speed of light.

④ $\vec{\nabla}\times\left(\mu_0\vec{H} + \mu_0\vec{M}\right) = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial}{\partial t}\left(\frac{1}{\epsilon_0}\vec{D} - \frac{1}{\epsilon_0}\vec{P}\right)$

$\mu_0\vec{\nabla}\times\vec{H} + \mu_0\vec{J}_b = \mu_0\vec{J} + \mu_0\frac{\partial}{\partial t}\vec{D} - \mu_0\frac{\partial\vec{P}}{\partial t}$

$\vec{\nabla}\cdot\vec{J}_p = -\frac{\partial\rho_b}{\partial t} = -\frac{\partial}{\partial t}\left(-\vec{\nabla}\cdot\vec{P}\right)$

$\Rightarrow \vec{\nabla}\cdot\left(\vec{J}_p - \frac{\partial\vec{P}}{\partial t}\right) = \phi$

$\Rightarrow J_p = \frac{\partial\vec{P}}{\partial t}$ $\{\vec{P}$ has no curl$\}$

$\vec{\nabla}\times\vec{H} = \left(\vec{J} - \vec{J}_b - \vec{J}_p\right) + \frac{\partial\vec{D}}{\partial t}$

$\vec{\nabla}\times\vec{H} = \vec{J}_f + \frac{\partial\vec{D}}{\partial t}$