PH320 exam 1: 2/6/08

Do any four (and **only** four) of the following questions.
No calculators. Closed book, although you may use a single crib sheet (Letter or A4 size).

1.
   - What does it mean for an operator to be Hermitian?
   - List three mathematical properties that make Hermitian operators fundamental in quantum mechanics.
   - Prove that the momentum operator is Hermitian.

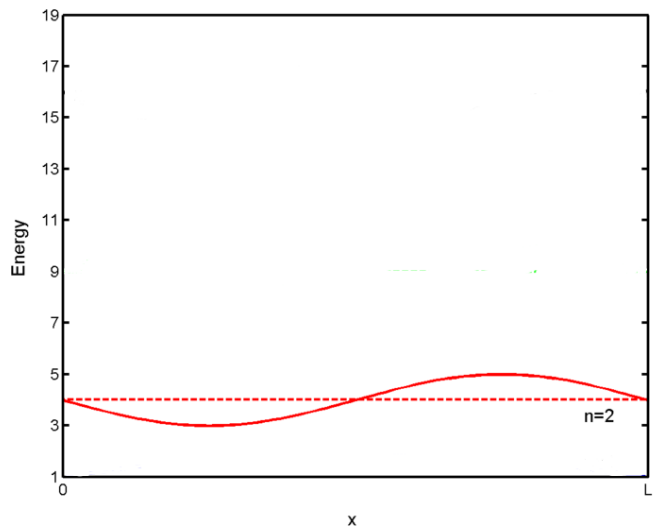
2. This problem requires some pencil and paper calculations. Feel free to round aggressively. I'll accept anything within a factor of 10.

   Let's model the Hydrogen-Chloride (HCl) molecule as a harmonic oscillator. In fact the Cl is so heavy compared to the H, that it is effectively at rest. Further, the mass of the hydrogen is essentially just mass of the proton, which is 1.67×10^{-27} kg. The hydrogen atom bounces back and forth like a ball on a spring.

   (a) Measurements indicate that the "spring constant" for HCl is 481 N/m. Using this spring constant, at what frequency (in Hz) does the HCl molecule vibrate when in its ground state. For this calculation, you can round k to 500 and m to 1.5×10^{-27}. (Mind factors of 2π!).

   (b) How much energy (in eV) would be required to excite this molecule to its first excited state? $\hbar = 1 \times 10^{-34} m^2 kg/s$. 1 J is 6×10^{18} eV.

3. The figure below is for a particle in an infinite square well potential. Use units in which $\pi^2\hbar^2/(2ma^2) = 1$. I've drawn in the first excited state. The dotted line refers to the energy and the solid line is the wavefunction. In like fashion, complete the figure for $n = 1, 3, 4$.



4. (a) Write down the ground state wavefunction of the harmonic oscillator.
   (b) Show explicitly that the lowering operator a applied to this function is exactly zero.

5. Let $\Psi(x,t) = Ae^{-a[mx^2/\hbar + it]}$.
   (a) Normalize $\Psi(x,t)$
   (b) Show that $\Psi(x,t)$ is a minimum uncertainty state; i.e., show that $\sigma_x \sigma_p = \hbar/2$.

6. A particle of mass $m = 1$ is placed in an infinite square well potential of width $a = 1$. It is initialized in a superposition of the ground state and the first excited state chosen so that there is a 90% probability that a measurement of the eneregy would yield $\pi^2\hbar^2/2$. Write down an expression for $\Psi(x,t)$.

7. Prove that the eigenstates of the harmonic oscillator Hamiltonian are also eigenstates of the operator $a^\dagger a$

8. Compute the following two commutators:
   (a) $[H, x]$
   (b) $[a, a^\dagger]$