HW-3

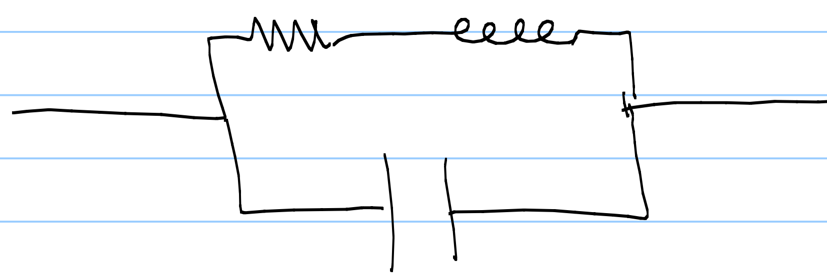
 9/27/2007

$$\hat{x} = \frac{F/m}{(\omega_0^2-\omega^2)+i\gamma\omega} = \frac{F/m}{\rho^2}$$

$$\rho^2 = \underbrace{\sqrt{(\omega_0^2-\omega^2)^2+\gamma^2\omega^2}}_{\frac{1}{x_0}} \, e^{\underbrace{i\tan^{-1}\left(\frac{\gamma\omega}{\omega_0^2-\omega^2}\right)}_{\varphi}}$$

Boas ch 2 16

8)



$$V_{RL} = V_R + V_L = (R + i\omega L) I_0 e^{i\omega t}$$

$$V_C = \frac{1}{i\omega C} I_0 e^{i\omega t}$$

$$V_{RLC} = Z I = \underbrace{\left[\frac{1}{V_{RL}} + \frac{1}{V_C}\right]^{-1}}_{Z} I$$

$$Z = \left[\frac{1}{R+i\omega L} + i\omega C\right]^{-1}$$

$$Z = \left[\frac{1+(R+i\omega L)\, i\omega C}{R+i\omega L}\right]^{-1}$$

$$= \frac{R+i\omega L}{\underbrace{(1-\omega^2 LC)}_{\equiv \alpha} + iR\omega C}$$

$$= \frac{R + i\omega L \quad (\alpha - iR\omega C)}{(\alpha + iR\omega)\,(\alpha - iR\omega C)}$$

$$= \frac{R\alpha + i\alpha\omega L - iR^2\omega C + R\omega^2 LC}{\alpha^2 + R^2\omega^2 C}$$

$$= \frac{R(\overbrace{\alpha}^{=1-\omega^2 LC} + \omega^2 LC) + i[\alpha\omega L - R^2\omega C]}{\alpha^2 + R^2\omega^2 C^2}$$

$$Z = \frac{R + i[(1-\omega^2 LC)\omega L - R^2\omega C]}{(1-\omega^2 LC)^2 + R^2\omega^2 C^2}$$

$$= \frac{R - i[\omega C R^2 + \omega^3 L^2 C - \omega L]}{(1-\omega^2 LC)^2 + (R\omega C)^2}$$

At resonance $\operatorname{Im}(Z) = 0$

So $Z = \dfrac{R}{(1-\omega^2 LC)^2 + (R\omega C)^2}$

But at resonance $\operatorname{Im}(Z) = 0$

$$\Rightarrow \omega C R^2 + \omega^3 L^2 C - \omega L = 0$$

$$\Rightarrow \omega^2 L^2 C = L - CR^2$$

$$\Rightarrow \omega^2 = \frac{L - CR}{L^2 C} = \frac{1}{LC} - \frac{R}{L^2}$$

So you plug this into

16.9 if $\operatorname{Arg}(z) = \pi/4 = 45°$

then $\operatorname{Im}(z) = \operatorname{Re}(z)$

From Eq. 16.15

$$R = \omega L - \frac{1}{\omega C} \Rightarrow$$

$$\omega C R = \omega^2 L C - 1$$

$$\Rightarrow \quad \omega^2 L C - \omega C R - 1 = 0$$

$$\Rightarrow \quad \omega^2 - \omega \frac{R}{L} - \frac{1}{LC} = 0$$

$$\Rightarrow \quad \omega = \frac{\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}}{2}$$

16.12 For any complex # z

$$|z|^2 = z\bar{z} = \underbrace{Im(z)^2 + Re(z)^2}_{\text{Pythagorous}}$$

So

$$\left|\sum_{N=0}^{\infty} r^{2N} e^{iN\theta}\right|^2 =$$

$$\underbrace{\left(\sum_{N=0}^{\infty} r^{2N} \cos N\theta\right)^2}_{Re} + \left(\sum_{N=0}^{\infty} r^{2N} \sin N\theta\right)^2$$

Im

Now $$(r^2 e^{i\theta})^N = r^{2N} e^{iN\theta}$$

$$\equiv \rho^N \quad \text{where}$$

$$\rho = r^2 e^{i\theta}$$

So $\sum_{n=0}^{\infty} r^{2n} e^{in\theta} = \sum_{n=0}^{\infty} \rho^n$

$= \lim_{N \to \infty} S_N$, where

$$S_N = \frac{1-\rho^N}{1-\rho}$$

$$|\rho| = |r^2 e^{i\theta}| \le |r|^2$$

So if $|r| < 1$ then

$$S_N \to S = \frac{1}{1-\rho}$$

$$= \frac{1}{1-r^2e^{i\theta}}$$

what we need is

$$\left|\frac{1}{1-r^2e^{i\theta}}\right|^2 = \frac{1}{(1-r^2e^{i\theta})(1-r^2e^{-i\theta})}$$

$$= \frac{1}{1-r^2\,2\cos\theta+r^4}$$ ✓

17.1 $\left(\frac{1+i}{1-i}\right)^{2718}$

$$1+i = \sqrt{2}\, e^{i\pi/4}$$

$$1-i = \sqrt{2}\, e^{-i\pi/4}$$

$$\Rightarrow \left(\frac{\sqrt{2}\, e^{i\pi/4}}{\sqrt{2}\, e^{-i\pi/4}}\right)^{2718}$$

$$= \left(e^{i\pi/2}\right)^{2718}$$

$$= e^{i\,1359\,\pi} = -1$$

17.2 $$\left(\frac{1+i\sqrt{3}}{\sqrt{2}(1+i)}\right)^5$$

$$1+i\sqrt{3} = 2e^{i\tan^{-1}\sqrt{3}} = 2e^{i\pi/3}$$

$$\sqrt{2}(1+i) = \sqrt{2}\cdot\sqrt{2}e^{i\pi/4}$$

$$= \left(e^{i\pi/12}\right)^5 = e^{i5\pi/12}$$

turns out $\cos(5\pi/12) = \frac{\sqrt{3}-1}{2\sqrt{2}}$

$\sin(5\pi/12) = \frac{\sqrt{3}+1}{2\sqrt{2}}$

17.3 $\sqrt[5]{-4-4i}$

$$-4-4i = -4(1+i)$$
$$= -4\sqrt{2}\, e^{i\pi/4}$$
$$= 4\sqrt{2}\, e^{-i\pi/4}$$

So the 5 roots have Args

$$\frac{\frac{7}{2}\pi}{5} = \frac{7}{10}\pi\ ; \ \frac{14}{10}\pi, \frac{21}{10}\pi, \frac{28}{10}\pi, \frac{35}{10}\pi$$

$$(4\sqrt{2})^{1/5} = \sqrt{2} \quad \left\{\underbrace{\sqrt{2}\sqrt{2}\sqrt{2}\sqrt{2}}_{4}\sqrt{2}\right\}$$

So we have the 5 roots

$$\sqrt{2}\left\{ e^{i7\pi/10}, e^{i14\pi/10} \ \ldots \ e^{i35\pi/10} \right\}$$

$\sinh(1+i\pi/2)$

$$\sinh(z) = \frac{e^{z} - e^{-z}}{2}$$

$$\Rightarrow \sinh(1+i\pi/2) = \frac{e^{(1+i\pi/2)} - e^{-(1+i\pi/2)}}{2}$$

$$= \frac{1}{2}\left[e\,\underbrace{e^{i\pi/2}}_{i} - e^{-1}\,\underbrace{e^{-i\pi/2}}_{-i}\right]$$

$$= \frac{i}{2}\left(e + 1/e\right) \approx i\,1.543$$

$\mathrm{Tanh}(i\pi/4)$

$$\sinh(i\pi/4) = \frac{e^{i\pi/4} - e^{-i\pi/4}}{2}$$

$$= i\sin(\pi/4)$$

$$= i\sqrt{2}/2$$

$$\cosh(i\pi/4) = \cos(\pi/4)$$

$$= \sqrt{2}/2$$

$$\underline{\tanh(i\pi/4) = i}$$

so $e^{x(1+i)} = e^{ax}$

$$= 1 + ax + \frac{a^2x^2}{2} + \cdots$$

$a = 1+i = \sqrt{2}\, e^{i\pi/4}$

so $e^{x(1+i)} = 1 + \sqrt{2}\, e^{i\pi/4} x + 2 e^{i\pi/2} \frac{x^2}{2}$

$+ \cdots$

N-th coeff is $\dfrac{2^{N/2} e^{iN\pi/2}}{N!}$

$$\boxed{e^{x(1+i)} = \sum_{N=0}^{\infty} \frac{2^{N/2} e^{iN\pi/2}}{N!} x^N}$$

Now $e^{x(1+i)} = e^{x} e^{ix}$

$$= e^{x}\left[\cos x + i \sin x\right]$$

So take Real & Imag parts of 🚩 gives:

$$e^x \cos x = \sum_{n=0}^{\infty} \frac{2^{n/2} \cos(n\pi x/2)}{n!} x^n$$

$$e^x \sin x = \sum_{n=0}^{\infty} \frac{2^{n/2}}{n!} \sin(n\pi x/2)\, x^n$$