Homework 1 — due 23 Oct. 2006 in class

PH507 Electrodynamics — posted: 2 October 2006

Reading: Heald and Marion (HM) chapter 14 and posted notes.

1) *Walk off in a birefringent crystal.* A quartz plate has thickness d and its optic axis makes an angle of 45^o to its faces. A ray of unpolarized light enters normal to the plate and leaves as two separate polarized rays. Given that no = 1.544 and ne = 1.533 for quartz, find the separation between the two exiting rays.

2) *Phase matching in birefringent nonlinear crystals.* In second harmonic generation, the conversion from the fundamental frequency ω_1 to the second harmonic $\omega_2 = 2\,\omega_1$ is most efficient if the phase-matching condition $\Delta k = k_2 - 2\,k_1 = 0$ is met. There are two common ways of doing this, creatively named Type I and Type II phase matching. In Type I phase matching, in a negative uniaxial crystal ($n_e < n_o$), the fundamental beam at ω_1 propagates with a polarization oriented along the ordinary axis, and the second harmonic propagates along with an angle to the optic axis (extraordinary axis) that is adjusted to satisfy the phase-matching condition. In beta-barium borate (BBO), the indices of refraction at 800nm are: $n_e = 1.54442$, $n_o = 1.66055$. At the harmonic wavelength of 400nm, they are $n_e = 1.56789$, $n_o = 1.69298$. Calculate the optimum phase matching angle such that $\Delta k = 0$.

3) In their rest frame, the mean lifetime of muons is 2μs. Muons are produced in the upper atmosphere, as cosmic ray secondaries.
   a. In observer's reference frame, the muons travel at 0.99c. Calculate the mean distance the muons will travel before decay using classical and relativistic approaches.
   b. What percentage of the muons produced at an altitude of 10km reach the ground, assuming they travel downward at 0.99c? (answer 9%)

4) HM problem 14-5. Hint: resolve the position vector **x** into components perpendicular and parallel to **v**. For the comparison, check the results to see if they produce the correct velocity addition formula for a boost parallel to the initial velocity.

5) HM problem 14-6.

6) A beam of electrons with an average energy of 1 MeV is collided with a laser beam with the aim of producing x-rays or γ-rays. By boosting to the reference frame of the electrons and back, calculate the Compton scattering (photon energy vs angle) for two cases
   a. Counter-propagating laser and electron beams
   b. Collision of the two beams at right angles.

   Comment on any differences you see.

7) HM problem 14-10