MATH 225 - Differential Equations September 16, 2008

Project 1, Fall 2008 **Due**: September 24, 2008

Lab 1.4 - Exponential and Logistics Population Models

For this project we consider lab 1.4 of Differential Equations pages 148 to 149. After reading this material construct a lab report addressing each of the following questions considering the states of New York and Hawaii:[1]

1. Assuming that the rate of growth is proportional to the amount populating we establish the following model equation,

$$\frac{dp}{dt} = kp, \quad k \in \mathbb{R}^+. \tag{1}$$

   (a) Construct a list of variables and parameters associated with (1) and describe the meaning of each.

   (b) Analytically solve (1) using the methods discussed in section 1.2 of the text.

   (c) Discuss qualitative behavior of the solutions to (1) through the equation's:

      i. Equilibrium Points

      ii. Phase Line

   (d) Given a mathematical model it is common to use recorded data to estimate values for unknown parameters within the model. In is not always obvious how one can best fit the parameters of a continuous problem to discrete data. There is inherent error both in the continuous approximation and data acquisition methods. Using the exponential growth model (1) and the data in table 1.11 on page 148 try to estimate the growth parameter k in various ways.

      i. Using the model and considering the derivative $\frac{dp}{dt}$ as a finite difference $\frac{\triangle p}{\triangle t} = \frac{p_2 - p_1}{t_2 - t_1}$ where $\triangle t$ is assumed to be ten years, approximate the growth parameter k in terms of known data.

      ii. Using your analytic solution from 1(b) approximate k the given data calculated data.

      iii. Using a graph of an Euler's method solution, pick you growth parameter k to fit your model to the data compiled in 1(d).

   (e) After you construct your growth parameter k use your model to complete problem one of the lab.

   Hint: The term linear extrapolations means using a linear approximation based on the data from 1990 and 2000. Construct the line through those two points and use the line to approximate p in 2010.

2. Assuming now a logistics model of growth we establish the following model equation,

$$\frac{dp}{dt} = kp\left(1 - \frac{p}{N}\right), \quad k, N \in \mathbb{R}^+. \tag{2}$$

   (a) Given (2) find the explicit solution, $p(t)$.

   (b) Using previous ideas approximate the carrying capacity N and k using only one of the methods from 1(d) .

   (c) Complete question two of the lab on page 149.

3. In a short essay format summarize your results from the previous questions. After your summary address the following questions:

   - Compare and contrast the approximations made by your models. Is one better than another? Are there particular scales where the approximations were good? What confidence can you place on the long-term predictions of your models.
   - What might you do to improve the validity of the models. Be sure to discuss how the model might change and what data requirements might be needed.

[1] Your report should be well organized and clearly presented. If steps are unclear then include more steps or make annotations clarifying the procedure and purpose. Be sure to label and title any included graphs or tables of data.