

Sketch of Schrödinger

Experiments such as electron diffraction suggested particles had wave-like properties. Hence de Broglie's

$$\lambda = h/p \quad \text{or} \quad \vec{k} = \vec{p}/\hbar$$

The simplest case to consider is a part. with a well defined momentum as a plane wave:

$$\psi(\vec{r}, t) = \psi_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

Since $\vec{k} = \vec{p}/\hbar$ de Broglie

and $\hbar\omega = E$ Einstein
 $= p^2/2m$ for free particles

$$\psi \text{ becomes } \psi(\vec{r}, t) = \psi_0 e^{i(\vec{p} \cdot \vec{r} - Et)/\hbar}$$

with $E = p^2/2m$

You can easily show that

$$\text{satisfies } i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi$$

$$\psi(\vec{r}, t) = \psi_0 e^{i(\vec{p} \cdot \vec{r} - Et)/\hbar}$$

$$\nabla^2 \psi = \frac{i^2}{\hbar^2} p^2 \psi = -\frac{2mE}{\hbar^2} \psi$$

$$\text{So } -\frac{\hbar^2}{2m} \nabla^2 \psi = E \psi$$

$$i\hbar \frac{\partial \psi}{\partial t} = i\hbar \left(-\frac{iE}{\hbar}\right) \psi = E \psi$$