MATH 225 - Differential Equations May 29 , 2008
Homework 5, Field 2008 **Due Date**: June 3, 2008

Linear Equations - Complex/Repeated/Zero Eigenvalues - Trace Determinant Plane

1. Given that $\frac{d\mathbf{Y}}{dt} = \mathbf{AY}$ where $\mathbf{A} = \begin{bmatrix} -2 & -3 \\ 3 & -2 \end{bmatrix}$.
   (a) Find the real valued general solution of this system.
   (b) Using HPGSystemSolver plot the phase portrait and classify the equilibrium solution.

2. Given that $\frac{d\mathbf{Y}}{dt} = \mathbf{AY}$ where $\mathbf{A} = \begin{bmatrix} -3 & 10 \\ -1 & 3 \end{bmatrix}$.
   (a) Find the real valued general solution of this system.
   (b) Using HPGSystemSolver plot the phase portrait and classify the equilibrium solution.

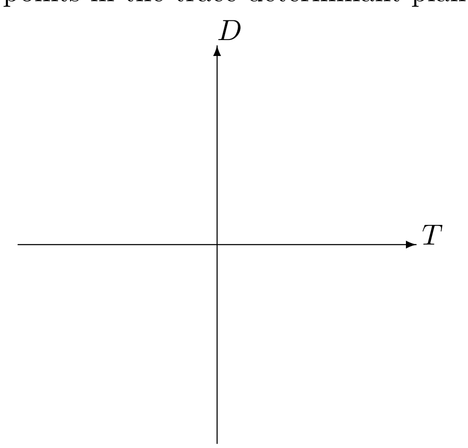
3. Given that $\frac{d\mathbf{Y}}{dt} = \mathbf{AY}$ where $\mathbf{A} = \begin{bmatrix} -3 & 1 \\ 3 & -1 \end{bmatrix}$.
   (a) Find the general solution of this system.
   (b) Using HPGSystemSolver plot the phase portrait and classify the equilibrium solutions.

4. Given that $\frac{d\mathbf{Y}}{dt} = \mathbf{AY}$ where $\mathbf{A} = \begin{bmatrix} -5 & 1 \\ -1 & -3 \end{bmatrix}$.
   (a) Find the solution of this system assuming that $\mathbf{Y} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$.
   (b) Using HPGSystemSolver plot the phase portrait and classify the equilibrium solution.

5. Given the general two-dimensional autonomous system of linear ODE's,

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}, \quad (1)$$

draw and label all off-axis equilibrium points in the trace-determinant plane.



Using the program TDAnimation discuss the changes to both the number and classification of equilibrium points as the TD-plane is traversed by the program.

HW6 Soln. (Abbreviated)

1. $A = \begin{bmatrix} -2 & -3 \\ 3 & -2 \end{bmatrix}$

$$\det(A-\lambda I) = (-2-\lambda)^2 + 9 = \lambda^2 + 4\lambda + 4 + 9 =$$
$$= \lambda^2 + 4\lambda + 13 = 0$$
$$\Rightarrow \lambda = -2 \pm 3i$$

$[A-\lambda I]\vec{v} = \vec{0} \Rightarrow \quad -3i\, v_1 = 3v_2 \Rightarrow \vec{v}^{(1)} = \begin{bmatrix} 1 \\ -i \end{bmatrix}, \ \vec{v}^{(2)} = \begin{bmatrix} 1 \\ i \end{bmatrix}$

Real Valued Soln.

$$\vec{Y}(t) = k_1 e^{-2t} \begin{bmatrix} \cos(3t) \\ \sin(3t) \end{bmatrix} + k_2 e^{-2t} \begin{bmatrix} \sin(3t) \\ -\cos(3t) \end{bmatrix}$$

2) $\det\left(\begin{bmatrix} -3-\lambda & 10 \\ -1 & 3-\lambda \end{bmatrix} \right) = (3-\lambda)(-3-\lambda) + 10 =$

$$= \lambda^2 \quad -9 + 10 = 0$$
$$\Rightarrow \lambda = \pm i$$

$[A-\lambda I]\vec{v} = \vec{0} \Rightarrow (-3 + -i)\, v_1 + 10 v_2 = 0 \quad \Rightarrow v_1 = 10,\ v_2 = 3+i$

$\Rightarrow \vec{v}^{(1)} = \begin{bmatrix} 10 \\ 3+i \end{bmatrix} \quad \vec{v}^{(2)} = \begin{bmatrix} 10 \\ 3-i \end{bmatrix}$

Real Valued Soln

$$\vec{Y}(t) = k_1 \begin{bmatrix} 10\cos(3t) \\ 3\cos(3t) - \sin(3t) \end{bmatrix} + k_2 \begin{bmatrix} 10\sin(3t) \\ 3\sin(3t) + \cos(3t) \end{bmatrix}$$

3) (a) $\det(A-\lambda I) = \det\left(\begin{bmatrix} -3-\lambda & 1 \\ 3 & -1-\lambda \end{bmatrix}\right) =$

$$= (-3-\lambda)(-1-\lambda) - 3 = \lambda^2 + 3\lambda + \lambda + 3 - 3 =$$
$$= \lambda^2 + 4\lambda = \lambda(\lambda+4) = 0$$
$$\lambda_1 = 0 \ , \quad \lambda_2 = -4$$

$[A-\lambda_1 I]\vec{v} = \vec{0} \Rightarrow 3v_1 = v_2 \Rightarrow \vec{v}^{(1)} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$[A-\lambda_2 I]\vec{v} = \vec{0} \Rightarrow \vec{v}^{(2)} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

General Soln:

$$\vec{Y}(t) = k_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{0t} + k_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-4t}$$

4) $\det(A-\lambda I) = (-5-\lambda)(-3-\lambda) + 1 =$
$$= \lambda^2 + 16 + 8\lambda = (\lambda+4)^2 = 0$$
$$\lambda = -4$$
$[A-\lambda I]\vec{v} = \vec{0} \Rightarrow \vec{v}^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

General Soln:
$$\vec{Y}(t) = \left(\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + t\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) e^{-4t}$$

5) See Text or Class notes for T-D diagram.
Also see T-D animation.