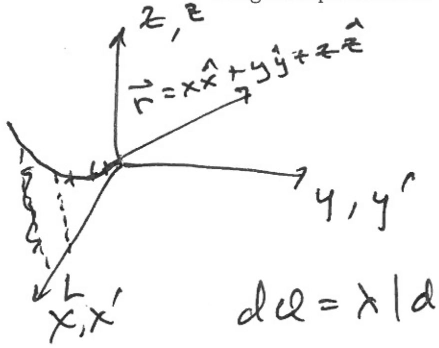
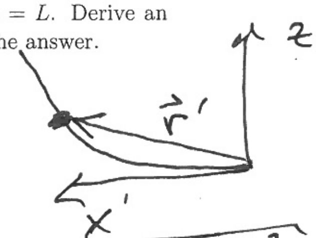
Exam 1 **Jan. 20** Name KEY

1. Charge is uniformly distributed on a non conducting wire in the shape of a parabola in the x-z plane. The equation determining this shape is $z = x^2$ with charge going from $x = 0$ to $x = L$. Derive an integral expression for $\vec{E}$ at an arbitrary point from which Mathematica will yield the answer.





$$d\vec{E} = \frac{k\,dq}{\mathscr{r}^2}\hat{\mathscr{r}}$$

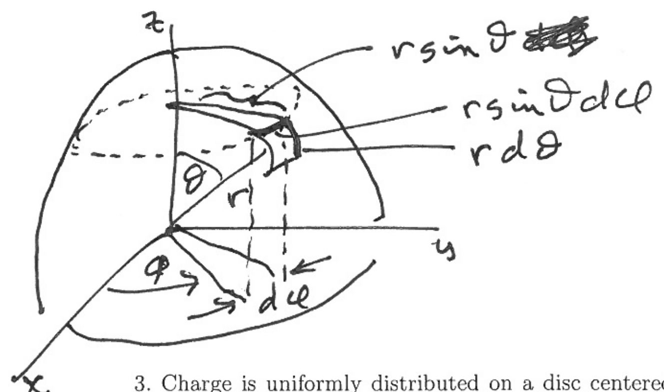
$$\vec{r}' = x'\hat{x} + x'^2\hat{z} + 0\,\hat{y}$$

$$d\vec{r}' = dx'\hat{x} + 2x'dx'\hat{z} \qquad |d\vec{r}'| = dx'\sqrt{1+4x'^2}$$

$$dq = \lambda|d\vec{r}'| = \lambda\,dx'\sqrt{1+4x'^2}$$

$$\vec{E} = \int d\vec{E} = \int_0^L \frac{k\lambda\,dx'\sqrt{1+4x'^2}}{\mathscr{r}^2}\hat{\mathscr{r}}$$

2. Derive an expression for the infinitesimal surface area (or tile) placed on the surface of a sphere of radius R. Write an integral from this with limits to determine the surface area of this sphere.



$$\int_0^{2\pi}\int_0^{\pi} r\sin\theta\,d\varphi\,r\,d\theta = \text{Vol}$$

3. Charge is uniformly distributed on a disc centered in the x-y plane of radius R. Derive an integral expression for $\vec{E}$ at an arbitrary point from which Mathematica will yield the answer. Write on the back of this page.

over