Name .....

Quiz 3
PH361

1. Write the divergence theorem for the vector function $\vec{\mathbf{E}}$.

$$\int_{Vol} \vec{\nabla}\cdot\vec{E}\, d\tau = \oint \vec{E}\cdot d\vec{a}$$

2. Write Stokes theorem for the vector function $\vec{\mathbf{E}}$.

$$\int \vec{\nabla}\times\vec{E}\cdot da = \oint \vec{E}\cdot d\vec{\ell}$$

3. Write an integral expression for the line integral of the vector function $\vec{v} = 6\hat{x} + yz^2\hat{y} + 3(y+z)\hat{z}$ along the y axis (z=0 to x=0) from 0 to 1.

$d\vec{r} = d\vec{\ell} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$ evaluate at $x=0$; $z=0$; $dx = dz = 0$

$$\int \vec{E}\cdot d\vec{\ell} = \int \left(6\hat{x} + yz^2\hat{y} + 3(y+z)\hat{z}\right)\cdot\left(dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}\right)$$

$$= \int_0^1 yz^2\, dy = 0 \quad (z = 0)$$

4. Write an integral expression for the energy stored in a uniformly charged sphere of radius R and charge q using the energy density $\varepsilon_0 E^2/2$ (use the appropriate values for E).

$$W = \frac{\epsilon_0}{2}\int E^2\, d\tau = \frac{\epsilon_0}{2}\left[\int_0^R\int_0^{2\pi}\int_0^{\pi} (E_{in})^2 r^2\sin\theta\, d\theta\, d\phi\, dr + \int_R^{\infty}\int_0^{2\pi}\int_0^{\pi} (E_{out})^2 r^2\sin\theta\, d\theta\, d\phi\right]$$

$$E_{out} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \qquad E_{in} = \frac{1}{4\pi\epsilon_0}\, q\frac{r}{R^3}$$

q is total charge of sphere