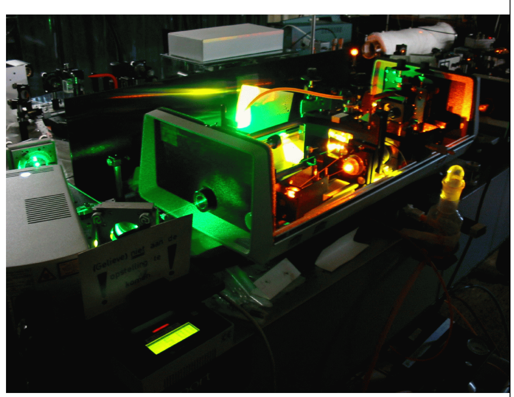
7

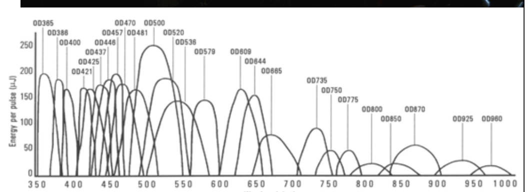
NL mixing solutions

SFG, DFG, SHG, OPA,…

Laser sources

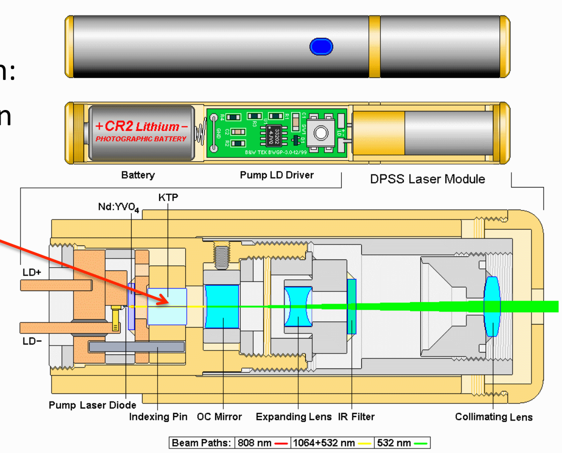
- Many applications (industrial, commercial, scientific) require affordable, efficient lasers at particular wavelengths.
- But lasers aren't available at all wavelengths.
- Laser wavelength chart
- Dye lasers can make most wavelengths:
- messy, not compact, unhealthy, hard to change dye
- Harmonic conversion and parametric amplification allow a solid-state alternative.





Intracavity doubling: Green laser pointer

- Pump: laser diode
- Laser: Nd:vanadate
- Frequency conversion:
- Intracavity doubling in KTP
- Type II allow use of both polarization components of IR
- HR reflects IR and green
- OC reflects most IR, passes green



Edmund Scientific L54-101 Green DPSS Laser Pointer

NL coupled equations for 2nd order mixing

- NL equations valid for
  - Define by choosing initial conditions, assumptions about which waves are the strongest

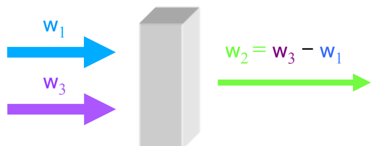
$$\frac{dA_1}{dz} = \frac{2i\,d_{eff}\,\omega_1^{\,2}}{k_1 c^2} A_3 A_2^{*} e^{-i\Delta k z} \qquad \frac{dA_2}{dz} = \frac{2i\,d_{eff}\,\omega_2^{\,2}}{k_2 c^2} A_3 A_1^{*} e^{-i\Delta k z}$$

$$\frac{dA_3}{dz} = \frac{2i\,d_{eff}\,\omega_3^{\,2}}{k_3 c^2} A_1 A_2 e^{+i\Delta k z} \qquad \Delta k = k_1 + k_2 - k_3$$

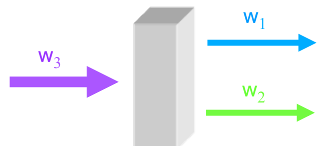
- What are the different ways we can use this 2nd order process to create new frequencies?

Difference-Frequency Generation: Optical Parametric Generation, Amplification, Oscillation

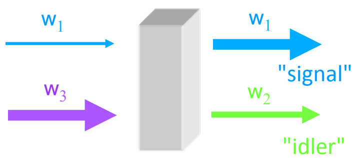
Difference-frequency generation takes many useful forms.



Parametric Down-Conversion
(Difference-frequency generation)

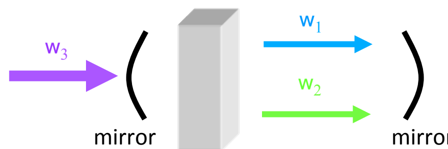


Optical Parametric Generation (OPG),
spontaneous down conversion



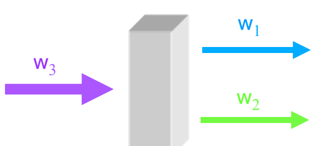
Optical Parametric
Amplification (OPA)

By convention:
$w_{signal} > w_{idler}$



Optical Parametric
Oscillation (OPO)

Optical Parametric Generation

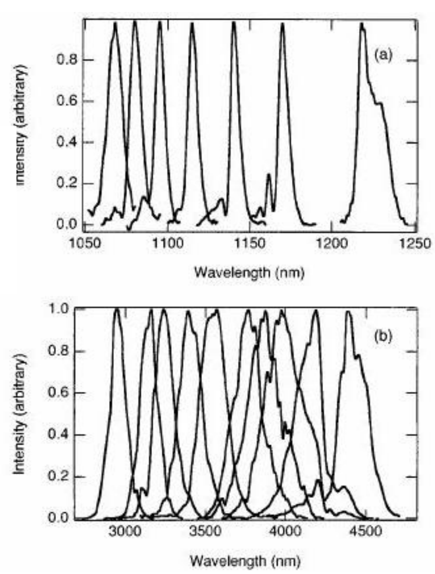


OPG:

- one strong pump
- spontaneous splitting of photon energy
- Amplification on the way out
- Tuning from phase matching angle

idler:

OPG with periodically poled $RbTiOAsO_4$



Sibbett, et al., Opt. Lett., **22**, 1397 (1997).

444 J. Opt. Soc. Am. B/Vol. 14, No. 2/February 1997

Ultrafast rainbow: tunable ultrashort pulses from a solid-state kilohertz system

Kent R. Wilson and Vladislav V. Yakovlev

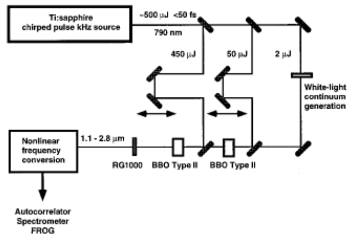


Fig. 3. Schematic of the experimental apparatus.

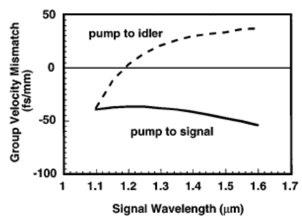


Fig. 1. Calculated group-velocity mismatch between pump and signal waves (solid curve) and pump and idler waves (dashed curve) for the 790-nm pump wavelength.

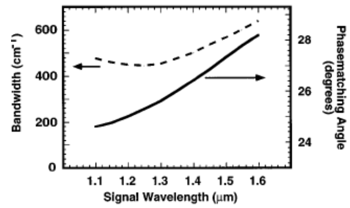


Fig. 2. Calculated external phase-matching angle (solid curve) and amplified bandwidth (dashed curve) for a 3-mm-long BBO crystal with a pump intensity of ~100 GW/cm^2.

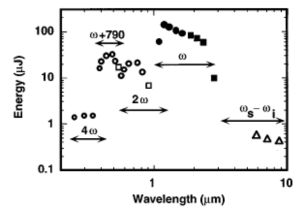
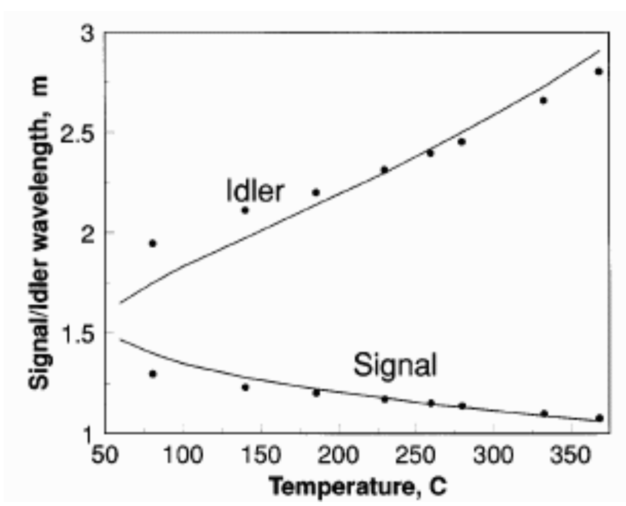


Fig. 7. Filled circles and filled squares show the measured energy of the amplified signal and idler waves, respectively. Open circles and open squares show the measured energy of the pulses generated by different nonlinear processes. 2ω, second harmonic of the signal (circles) and the idler (squares) (type I phase matching in 0.25-mm-thick BBO); $\omega + 790$: sum-frequency generation from the residual 790 nm and signal (type II phase matching in 0.2-mm-thick BBO); 4ω, fourth harmonic ($2\omega + 2\omega$) of the signal pulse (type II in 0.2-mm-thick BBO); open triangles, $\omega_S - \omega_i$: difference frequency generation (Type I in 1-mm-thick $AgGaS_2$).

Phase-matching in OPA

We can vary the crystal angle in the usual manner, or we can vary the crystal temperature (since n depends on T).

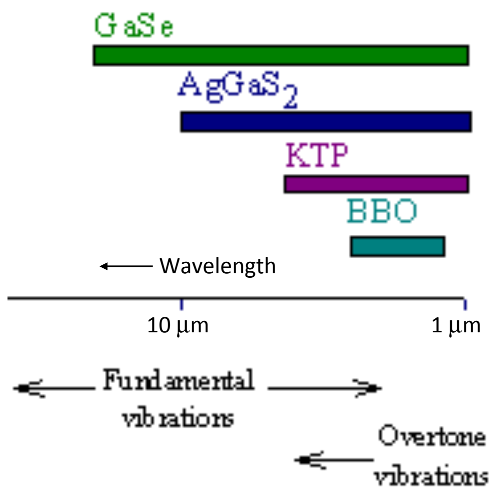


Difference frequency mixing for far-IR generation

Typically start with both outputs of an OPA.

With unusual crystals, such as $AgGaS_2$, $AgGaSe_2$ or GaSe, one can obtain radiation to wavelengths as long as 20 µm.

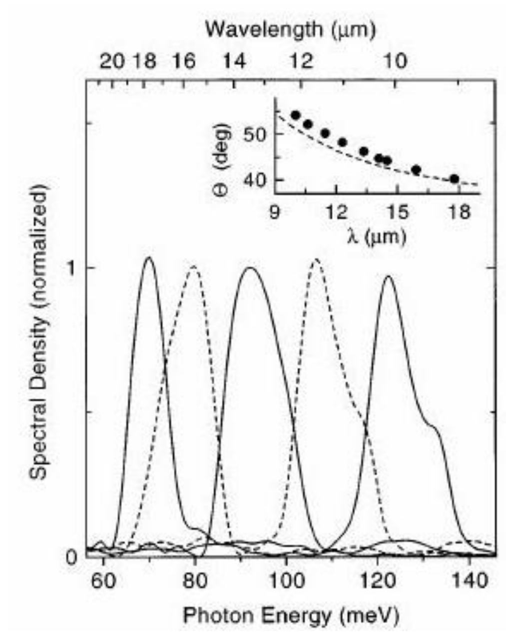
These long wavelengths are useful for vibrational spectroscopy.



Gavin D. Reid, University of Leeds, and Klaas Wynne, University of Strathclyde

Difference-frequency generation in GaSe

Angle-tuned wavelength



Elsaesser, et al., *Opt. Lett.*, **23**, 861 (1998)

NL coupled equations for 2nd order mixing

- NL equations valid for
  - Sum frequency mixing
  - Difference frequency mixing (DFG, OPA, OPO, SPDC)
  - Define by choosing initial conditions, assumptions about which waves are the strongest

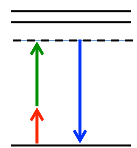
$$\frac{dA_1}{dz} = \frac{2i d_{eff} \omega_1^2}{k_1 c^2} A_3 A_2^* e^{-i\Delta k z}$$

$$\frac{dA_2}{dz} = \frac{2i d_{eff} \omega_2^2}{k_2 c^2} A_3 A_1^* e^{-i\Delta k z}$$

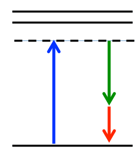
$$\frac{dA_3}{dz} = \frac{2i d_{eff} \omega_3^2}{k_3 c^2} A_1 A_2 e^{+i\Delta k z}$$

$$\Delta k = k_1 + k_2 - k_3 \qquad \omega_3 = \omega_1 + \omega_2$$

Sum frequency mixing

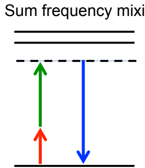


Difference frequency mixing



Simplify mixing equations

Sum frequency mixing



- Set up mixing equations

$$\omega_3 = \omega_1 + \omega_2 \qquad \Delta k = k_1 + k_2 - k_3$$

$$\frac{dA_1}{dz} = \frac{2i d_{eff} \omega_1^2}{k_1 c^2} A_3 A_2^* e^{-i\Delta k z} = i\xi \frac{\omega_1}{n_1} A_3 A_2^* e^{-i\Delta k z}$$

$$\frac{dA_2}{dz} = \frac{2i d_{eff} \omega_2^2}{k_2 c^2} A_3 A_1^* e^{-i\Delta k z} = i\xi \frac{\omega_2}{n_2} A_3 A_1^* e^{-i\Delta k z}$$

$$\frac{dA_3}{dz} = \frac{2i d_{eff} \omega_3^2}{k_3 c^2} A_1 A_2 e^{+i\Delta k z} = i\xi \frac{\omega_3}{n_3} A_1 A_2 e^{+i\Delta k z}$$

$$\xi = \frac{2 d_{eff}}{c}$$

- Initial conditions determine which equations to use
- SFG example: Start with amplitude at A_1 and A_2, $A_3 = 0$

Upconversion

- Want to shift weak IR emission (λ_1) to visible frequency to make it easier to detect
- Mix with strong visible beam (λ_2) to upshift signal to visible
  - Ex: λ_1=2500nm, λ_2=532nm, λ_3= ?

Upconversion

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- Mix with strong visible beam (λ_2) to upshift signal to visible
  - Ex: λ_1=2500nm, λ_2=532nm, λ_3= 439nm

$$\frac{1}{\lambda_3} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$$

- Approximations:
  - Perfect phase-matching $\Delta k = 0$
  - Nondepleted pump $\frac{dA_2}{dz} = 0$

$$\frac{dA_1}{dz} = i\xi\frac{\omega_1}{n_1}A_3A_2^*e^{-i\Delta kz} = i\xi\frac{\omega_1}{n_1}A_2^*A_3 \qquad \frac{dA_3}{dz} = i\xi\frac{\omega_3}{n_3}A_1A_2$$

Combine 2 1st order equations to one 2nd order

$$\frac{d^2A_1}{dz^2} = i\xi\frac{\omega_1}{n_1}A_2^*\frac{dA_3}{dz} = \left(i\xi\frac{\omega_1}{n_1}A_2^*\right)\left(i\xi\frac{\omega_3}{n_3}A_1A_2\right)$$

Solutions for up converted signal

- Growth of signal (at shortest wavelength) follows SHO equation: $\frac{d^2A_1}{dz^2} = -\xi^2 \frac{\omega_1}{n_1}\frac{\omega_3}{n_3}|A_2|^2 A_1 = -\kappa^2 A_1$

$$\frac{d^2A_1}{dz^2} = -\kappa^2 A_1$$

Defining the growth coefficient:

$$\kappa^2 = \xi^2 \frac{\omega_1\omega_3}{n_1 n_3}|A_2|^2 = \frac{4d_{eff}^2\omega_1\omega_3}{n_1 n_3 c^2}|A_2|^2$$

κ is real and has dimensions 1/length

General form of solution for arbitrary initial conditions:

$$A_1(z) = B\cos\kappa z + C\sin\kappa z$$

$$A_1'(z) = -B\kappa\sin\kappa z + C\kappa\cos\kappa z = i\xi\frac{\omega_1}{n_1}A_2^* A_3$$

Solve for the wave that is being upconverted:

$$A_3(z) = \frac{1}{i\xi\frac{\omega_1}{n_1}A_2^*}\left(-B\kappa\sin\kappa z + C\kappa\cos\kappa z\right) = -i\frac{n_1}{\xi\omega_1 A_2^*}\left(-B\kappa\sin\kappa z + C\kappa\cos\kappa z\right)$$

Upconversion solutions

- Solution is valid for any initial condition, as long as A_2 is constant

$$A_1(z) = B\cos\kappa z + C\sin\kappa z \qquad A_3(z) = -i\frac{n_1}{\xi\omega_1 A_2^*}\left(-B\kappa\sin\kappa z + C\kappa\cos\kappa z\right)$$

- Apply initial conditions: for upconversion, no A_3 at input, weak A_1

$$A_1(z) = A_1(0)\cos\kappa z \qquad A_3(z) = i\,A_1(0)\frac{n_1\kappa}{\xi\omega_1 A_2^*}\sin\kappa z$$

$$|A_3(z)|^2 = |A_1(0)|^2\left|\frac{n_1\kappa}{\xi\omega_1 A_2^*}\right|^2\sin^2\kappa z = |A_1(0)|^2\left|\frac{n_1 c}{2d_{eff}\,\omega_1 A_2^*}\right|^2\frac{4d_{eff}^2\omega_1\omega_3}{n_1 n_3 c^2}|A_2|^2\sin^2\kappa z$$

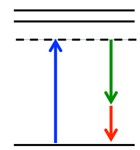
$$|A_3(z)|^2 = |A_1(0)|^2\frac{n_1\omega_3}{n_3\omega_1}\sin^2\kappa z$$

- Signal oscillates, so pick length (or pump intensity) carefully
- Max conversion corresponds to photon energy ratio

$$|A_3(z)|^2 \approx |A_1(0)|^2\frac{4d_{eff}^2\omega_3^2}{n_3^2 c^2}|A_2|^2 z^2 \quad \text{for small z}$$

Parametric amplification

Difference frequency mixing



- Difference frequency mixing, no phase mismatch
- Strong, constant pump A_3
- Weak seed at A_1 (Signal)

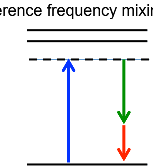
$$\omega_2 = \omega_3 - \omega_1 \qquad \Delta k = k_1 + k_2 - k_3$$

$$\frac{dA_1}{dz} = i\xi \frac{\omega_1}{n_1} A_3 A_2^* e^{-i\Delta k z} \qquad \frac{dA_2}{dz} = i\xi \frac{\omega_2}{n_2} A_3 A_1^* e^{-i\Delta k z} \qquad \frac{dA_3}{dz} = i\xi \frac{\omega_3}{n_3} A_1 A_2 e^{+i\Delta k z}$$

- Simplify equations, set up a 2nd order equation for $A_2(z)$

Parametric amplification

Difference frequency mixing



- Difference frequency mixing, no phase mismatch
- Strong, constant pump A_3

$$\omega_2 = \omega_3 - \omega_1$$

$$\Delta k = k_1 + k_2 - k_3 = 0$$

$$\frac{dA_3}{dz} = i\xi \frac{\omega_3}{n_3} A_1 A_2 e^{+i\Delta k z} = 0$$

$$\frac{dA_1}{dz} = i\xi \frac{\omega_1}{n_1} A_3 A_2^* e^{-i\Delta k z} \rightarrow i\xi \frac{\omega_1}{n_1} A_3 A_2^* \qquad \frac{dA_2}{dz} = i\xi \frac{\omega_2}{n_2} A_3 A_1^* e^{-i\Delta k z} \rightarrow i\xi \frac{\omega_2}{n_2} A_3 A_1^*$$

$$\rightarrow \frac{d^2 A_1}{dz^2} = i\xi \frac{\omega_1}{n_1} A_3 \frac{dA_2^*}{dz} \qquad \frac{dA_2^*}{dz} = -i\xi \frac{\omega_2}{n_2} A_3^* A_1$$

$$\frac{d^2 A_1}{dz^2} = \left(i\xi \frac{\omega_1}{n_1} A_3 \right)\left(-i\xi \frac{\omega_2}{n_2} A_3^* A_1 \right) = +\xi^2 \frac{\omega_1}{n_1} \frac{\omega_2}{n_2} |A_3|^2 A_1 = +\kappa^2 A_1$$

$$\frac{d^2 A_2}{dz^2} = +\kappa^2 A_2$$

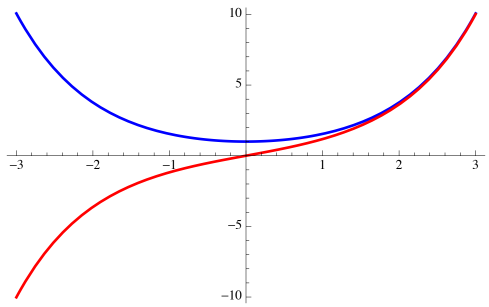
Because of positive sign, *real exponential solutions.*

Review: cosh, sinh

- Hyperbolic functions

$$\cosh(z) = \frac{1}{2}\left(e^{z} + e^{-z}\right) = \cos(iz) \qquad \sinh(z) = \frac{1}{2}\left(e^{z} - e^{-z}\right) = -i\sin(iz)$$

$$\frac{d}{dz}\cosh(az) = a\sinh(az) \qquad \frac{d}{dz}\sinh(az) = a\cosh(az)$$



Cosh(z) is even
Sinh(z) is odd

For large +z, both are exponentially increasing

Exponential gain in OPA

- Initial conditions: $A_1 = A_{10}$, no input at A_2

$$A_1(z) = A_1(0)\cosh\kappa z \qquad A_2(z) = C\sinh\kappa z + D\cosh\kappa z \rightarrow C\sinh\kappa z$$

$$\frac{dA_1}{dz} = A_1(0)\kappa\sinh\kappa z = i\xi\frac{\omega_1}{n_1}A_3A_2^* = i\xi\frac{\omega_1}{n_1}A_3C^*\sinh\kappa z$$

$$C = i\frac{n_1\kappa}{\xi\omega_1A_3}A_1(0) \qquad A_2(z) = i\frac{n_1\kappa}{\xi\omega_1A_3}A_1(0)\sinh\kappa z$$

- Both signal and idler have exponential gain instead of quadratic gain.
- For κz>>1, $I_1(z) \sim I_1(0)\exp[2\kappa z]$
- Presence of signal stimulates production of idler and vice-versa
- Very little net conversion if process is not phase-matched: crystal phase-matching angle tunes output

Gain coefficient

- Gain can be very high:

$$2\kappa = 2\times 2d_{eff}\sqrt{\frac{\omega_1\omega_2}{c^2 n_1 n_2}}|A_3| = 4d_{eff}\sqrt{\frac{\omega_1\omega_2}{c^2 n_1 n_2}}\sqrt{\frac{I_3}{2n_3\varepsilon_0 c}}$$

$$1\text{V/m} = \sqrt{377\,\text{W/m}^2}$$

$$1W/\text{m}^2 = 10000W/cm2 = \sqrt{\frac{1}{377}}V/m$$

- For BBO

$$2\kappa \approx 4d_{eff}\frac{2\pi}{n\lambda_p/2}\sqrt{\frac{I_3}{n\varepsilon_0 c/2}} = \frac{16\pi}{n}\frac{1}{\lambda_p}d_{eff}\sqrt{\frac{I_3}{2n\varepsilon_0 c}}$$

$$d_{eff} = 2pm/V$$

$$n = 1.6$$

$$\kappa \approx 7\times 10^{-4}\frac{\sqrt{I_3(W/cm^2)}}{\lambda_p(\mu m)}/cm$$

Exp[14]=10^6
For this single pass gain and L = 5mm, need
2κ =14/L =28 /cm
Need about I_3 = 2x10^9W/cm^2

Phase matching for parametric mixing

- For negative uniaxial: n_e is lowest, so place ω_3 as *e*-wave
- Type I: ω_1 and ω_2 parallel polarization, along *o*-direction
  - Generally broader bandwidth for OPA
- Type II: one of lower frequencies is along *e*-direction
  - can separate signal and idler with polarizer
- Quasi-phasematching (2.4), periodic poling

$$\Delta k = k_1 + k_2 - k_3 = \frac{1}{c}(\omega_1 n_1 + \omega_2 n_2 - \omega_3 n_3)$$

this generalizes to a vector relation

Note: for wave mixing, phase matching isn't just matching phase velocities or ref. indices. For SHG:

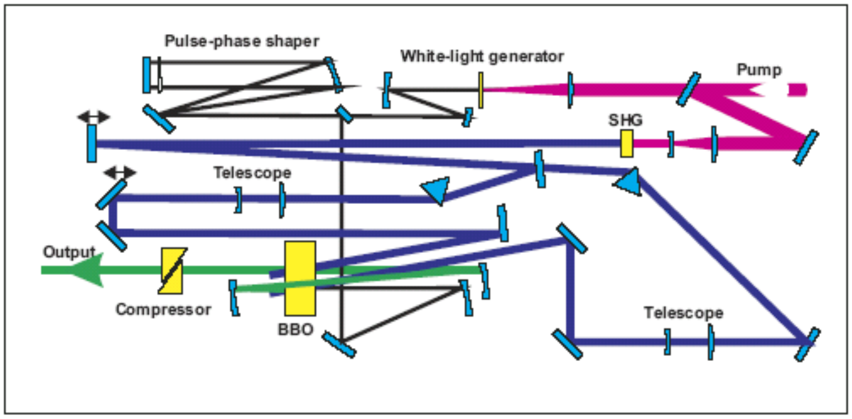
$$\Delta k = 2k_1 - k_2 = \frac{2\omega_1}{c}(n_1 - n_2)$$

An ultrafast noncollinear OPA (NOPA)

LIGHT CONVERSION

Continuum generates an arbitrary-color seed pulse.

TOPAS-white



Power conservation

- Manley-Rowe relations (2.5)

$$\frac{d}{dz}\left(\frac{I_1}{\omega_1}\right)=\frac{d}{dz}\left(\frac{I_2}{\omega_2}\right)=-\frac{d}{dz}\left(\frac{I_3}{\omega_3}\right)$$

- Can use these to reduce the number of coupled equations
- Helpful to understand saturated conversion limits in parametric processes:
  - $I/\hbar\omega$ = photon flux
  - For every photon taken from wave 3, we add one to waves 1 and 2
  - Example: can avoid back conversion in sum frequency generation only if there isn't any light left in both input beams

Scaling the NL equations

- Using scaled variables can simplify the NL equations and highlight the characteristic parameters
- SHG example $\omega_2 = 2\omega_1 \quad \Delta k = 2k_1 - k_2$ 2.7.10, 2.7.11

$$\frac{dA_1}{dz} = \frac{2\,i\,d_{eff}\omega_1^2}{k_1 c^2} A_2 A_1^* e^{-i\Delta k z} \qquad \frac{dA_2}{dz} = \frac{i\,d_{eff}\omega_2^2}{k_2 c^2} A_1^2 e^{+i\Delta k z}$$

- Scale the fields to the total intensity

$$I_j = 2n_j\varepsilon_0 c\left|A_j\right|^2 \qquad a_j = u_j e^{i\phi_j}$$

$$\left|a_j\right|^2 = u_j^2 = \frac{I_j}{I} = \frac{2n_j\varepsilon_0 c}{I}\left|A_j\right|^2 \qquad A_j = u_j\sqrt{\frac{I}{2n_j\varepsilon_0 c}}\,e^{i\phi_j}$$

Writing dimensionless equations

- Rewrite equations with new variables

$$\frac{dA_1}{dz} = \frac{2\,i\,d_{eff}\omega_1^2}{k_1 c^2} A_2 A_1^* e^{-i\Delta k z} \qquad A_j = \sqrt{\frac{I}{2n_j\varepsilon_0 c}}\,u_j e^{i\phi_j} = \sqrt{\frac{I}{2n_j\varepsilon_0 c}}\,a_j$$

$$\frac{d}{dz}\left(u_1\sqrt{\frac{I}{2n_1\varepsilon_0 c}}\,e^{i\phi_1}\right) = \frac{2\,i\,d_{eff}\omega_1}{n_1 c} u_2\sqrt{\frac{I}{2n_2\varepsilon_0 c}}\,e^{i\phi_2} u_1\sqrt{\frac{I}{2n_1\varepsilon_0 c}}\,e^{-i\phi_1} e^{-i\Delta k z}$$

$$\boxed{\frac{da_1}{dz} = i\frac{2d_{eff}\omega_1}{n_1 c}\sqrt{\frac{I}{2n_2\varepsilon_0 c}}\,a_2\,a_1^* e^{-i\Delta k z}}$$

$$\frac{dA_2}{dz} = \frac{i\,d_{eff}\omega_2^2}{k_2 c^2} A_1^2 e^{+i\Delta k z} \qquad \sqrt{\frac{I}{2n_2\varepsilon_0 c}}\frac{da_2}{dz} = i\frac{d_{eff}2\omega_1}{n_2 c}\frac{I}{2n_1\varepsilon_0 c}a_1^2 e^{+i\Delta k z}$$

$$\boxed{\frac{da_2}{dz} = i\frac{2d_{eff}\omega_1}{n_1 c}\sqrt{\frac{I}{2n_2\varepsilon_0 c}}\,a_1^2 e^{+i\Delta k z}} \qquad l = \frac{c}{2\omega_1 d_{eff}}\sqrt{\frac{2n_1^2 n_2\varepsilon_0 c}{I}}$$

Final form of scaled equations for SHG

$$\frac{da_1}{dz} = i\frac{1}{l} a_2 \, a_1^* e^{-i\Delta k z} \qquad \frac{da_2}{dz} = i\frac{1}{l} a_1^2 e^{+i\Delta k z} \qquad |a_1|^2 + |a_2|^2 = u_1^2 + u_2^2 = 1$$

- Define dimensionless distance variable

$$\xi = z / l$$

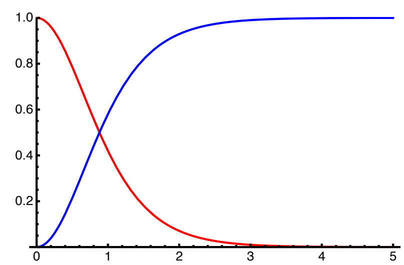
$$\Delta k \, z = \Delta k \, l \, \xi \equiv \Delta s \, \xi$$

$$\boxed{\frac{da_1}{d\xi} = i \, a_2 \, a_1^* e^{-i \Delta s \xi}} \qquad \boxed{\frac{da_2}{d\xi} = i \, a_1^2 e^{+i \Delta s \xi}}$$

- *l* is the characteristic distance for energy exchange (saturation and back-conversion)
- If there is no SH present at start, full conversion

Saturated SHG conversion

- No seed SH



- With seed