PHGN 462 Homework 1

1) Take a nice, long solenoid of radius *a* and coil density *n*. It's long enough that it's a pretty good approximation to call it infinite.

a) If there's some steady current I_0 running through it, what is the B-field inside and outside the solenoid? You don't have to derive it – just write it down.

b) Now suppose that that current is varying in time. In particular $I(t) = I_0 + kt$ for some positive constant *t*. Derive an expression for the E-field (if any) present in the system, both for $r < a$ and for $r > a$. Make sure you specify the direction of the field in addition to the magnitude.

c) Explicitly calculate the curl of your E-field inside and outside the solenoid and comment on what you get. Is it consistent with Faraday's law in differential form?

2) (from Pollack and Stump 10.34)

a) Show that the differential equation describing a simple RLC circuit driven by an AC voltage is:

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = V_0 \sin \omega t$$

Where Q is the charge on the capacitor and ω is the frequency of the driving voltage.

b) Derive the <u>steady state</u> solution *Q(t)* for this differential equation. Feel free to do it analytically or in Mathematica, but show the code if you do the latter. If you do use Mathematica, make sure the output you get is clear and interpretable, and not a six-page jumble of conditionals and such. Also, whatever you do, verify that in the limit as $R \to 0$, you get

$$Q(t) = \frac{V_0/L}{(\omega_0^2 - \omega^2)} \sin \omega t$$

Where $\omega_0 = \frac{1}{\sqrt{LC}}$, the resonant frequency for an RL circuit.

c) Now let $L = 1$, $C = 1$, and $R = 0.1$, for some arbitrary system of units. Solve our differential equation numerically, for initial values $Q(0) = 0$ and $I(0) = 0$, and driving frequency $\omega = 0.5$. You should see that after a few cycles, the solution settles down into an oscillation with the same frequency as the driving source.

d) Make plots of *Q(t)* for a few different ω's to show that for ω near ω_0, you get resonant behavior. Also plot your solution and the driving signal on the same axes and confirm that the driving signal and the response are out of phase with one another.

3) I think by now everyone has seen a Tesla coil. And, as it turns out, we now know enough to completely understand their operation. The most common design is basically a couple transformers and a capacitor (later designs involve vacuum tubes and transistors and such, but let's not worry about those).

Sketch out a design for a Tesla coil that'll make foot-long sparks when plugged into a standard 120V 60Hz wall outlet. Tell me what kind of inductors, capacitors, transformers, etc need to go in there, in which arrangement. I'd like a schematic and also a description of the parts and why they're there.

I'm deliberately leaving this rather open. It's a design task. That's kind of how those are. But correspondingly, I invite you to look up whatever you want in books or on the internet or wherever. Just get it done and tell me how.