

# H W6 SOLUTIONS

Note Title ch3 sec 9

10/8/2007

$$(AB)C \stackrel{?}{=} A(BC)$$

①

$$[(AB)C]_{i\bar{j}} = \sum_k (AB)_{ik} C_{kj}$$

$$= \sum_k \left( \sum_e A_{ie} B_{ek} \right) C_{kj}$$

$$= \sum_e \sum_k A_{ie} B_{ek} C_{kj}$$

$$= \sum_e A_{ie} \left( \sum_k B_{ek} C_{kj} \right)$$

sum doesn't depend  
on  $e$ .

$$= \sum_e A_{ie} (BC)_{ek}$$

$$= A(BC)$$

$$4) \begin{pmatrix} 0 & 2i & -1 \\ -i & 2 & 0 \\ 3 & 0 & 0 \end{pmatrix} \equiv A$$

$$A^T = \begin{pmatrix} 0 & -i & 3 \\ 2i & 2 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

$$\bar{A} = \begin{pmatrix} 0 & -2i & -1 \\ i & 2 & 0 \\ 3 & 0 & 0 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 0 & 0 & 1/3 \\ 0 & 1/2 & i/6 \\ -1 & i & -1/3 \end{pmatrix}$$

$$A^+ = \begin{pmatrix} 0 & i & 3 \\ -2i & 2 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

$$\textcircled{13} \quad A = \begin{pmatrix} (1 + i\sqrt{3})/4 & \frac{\sqrt{3}}{2\sqrt{2}}(1+i) \\ \frac{-\sqrt{3}}{2\sqrt{2}}(1+i) & (\sqrt{3} + i)/4 \end{pmatrix}$$

$$A^+ = \begin{pmatrix} (1 - i\sqrt{3})/4 & \frac{-\sqrt{3}}{2\sqrt{2}}(1-i) \\ \frac{\sqrt{3}}{2\sqrt{2}}(1-i) & (\sqrt{3} - i)/4 \end{pmatrix}$$

$$A^+ A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\underline{S_a} \quad \begin{matrix} A & B & C \\ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} & \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \end{matrix}$$

$$A^+ = A^T = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = A$$

$$B^+ = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = B$$

$$C^+ = C^T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = C \quad \checkmark$$

$$AB - BA = \begin{pmatrix} 2i & 0 \\ 0 & -2i \end{pmatrix}$$

$$BC - CB = \begin{pmatrix} 0 & 2i \\ 2i & 0 \end{pmatrix}$$

$$CA - AC = \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}$$

$$[C, [A, B]] = 0$$

$$[A, [B, C]] = 0$$

$$[B, [C, A]] = 0$$

20 Note

$$\det(AB) = \det(A) \det(B)$$

$$\text{So } \det(A^+ A) = \det(A^+) \det(A)$$

So if  $A^+A = I$  then

$$\text{Det}(A^+) \text{Det}(A) = \underline{1}$$

$$\begin{aligned} \text{now } \text{Det}(A^+) &= \overline{\text{Det}(A^T)} \\ &= \overline{\text{Det}(A)} \end{aligned}$$

$$\text{So } |\text{Det}(A)|^2 = 1$$

## Section 11

$$\textcircled{12} \quad M = \begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix}$$

$$C = \begin{bmatrix} 1 & -3 \\ 1 & 2 \end{bmatrix}$$

$$D = \begin{bmatrix} 4 & 0 \\ 0 & -1 \end{bmatrix}$$

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$$\textcircled{20} \quad M = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & -1 & -1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$D = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$$