

4. (10 pts) A square wire of side L with current I_0 is located symmetrically about the origin with its sides aligned with the xy coordinates. Find an expression for the vector potential far from the loop.

Principles

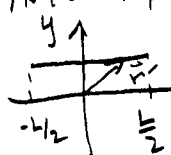
Method

Check

Defn:

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{I d\vec{\ell}}{r}$$

Find $r = |\vec{r}| = |\vec{r} - \vec{r}'|$ where $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$
 use CARTESIAN UNIT VECTORS FOR INTEGRAL SINCE THEY COME OUTSIDE. BREAK INTEGRAL INTO 4 parts one for each side.



$\vec{r}' = \frac{L}{2}\hat{y} + x\hat{x}$ $d\vec{r}' = d\vec{\ell} = dx\hat{x}$
 Integrate $\frac{\mu_0}{4\pi} \int_{-L/2}^{L/2} \frac{I d\vec{\ell}}{r}$

If $L \rightarrow 0$
 then $A \rightarrow 0$
 If $r \rightarrow \infty$
 then $A \rightarrow 0$
 If $I \rightarrow 0$
 then $A \rightarrow 0$

5. (10 pts) Show that a free electron could or could not be accelerated while confined to circular motion in a solenoid with slowly varying current $I(t)$.

Principles

Method

Check

$$\vec{F} = q\vec{v} \times \vec{B} + q\vec{E} = m\vec{a}$$

E from Faraday's Law
 B from Ampere's Law

Find \vec{E} from Faraday's Law $\oint \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{a}$
 where B comes from Ampere's Law applied to the solenoid. Put into Newton's 2nd using the Lorentz force.

$$q\vec{v} \cdot \hat{r} + qE\hat{\phi} = -\frac{mv^2}{r}\hat{r} + m\frac{dv}{dt}\hat{\phi}$$

coeff. of \hat{r} must be equal as well as $\hat{\phi}$ coeff.

$$q\vec{v} \cdot \hat{r} = -\frac{mv^2}{r} \quad qE = m\frac{dv}{dt}$$

Put in expressions for B & E . Then see if these two equations are consistent (They are not it turns out).

If $I(t) \rightarrow 0$
 then no motion of electron if it was at rest.