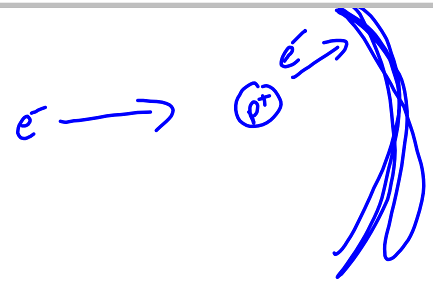
Reading: Today 10.3

Tomorrow 11.1



Scattering

The point is e^- are very much like point charges.

Aside: What is EM energy of a point charge?

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

$$u_{em} = \frac{\epsilon_0}{2}E^2 + \cancel{\frac{1}{2\mu_0}B^2} \xrightarrow{} 0 = \frac{\epsilon_0}{2}\frac{1}{16\pi^2\epsilon_0^2}\frac{q^2}{r^4}$$

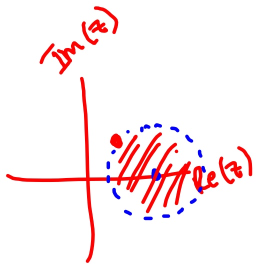
$$U_{em} = \int u_{em}\,dV = \iiint u_{em}\, r^2 \sin\theta \, d\theta \, d\phi \, dr$$

$$= \lim_{a\to 0}\int_a^\infty u_{em}\, 4\pi r^2 \, dr$$

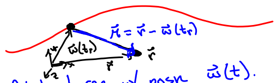
$$= \lim_{a\to 0}\int_a^\infty \frac{q^2}{8\pi\epsilon_0}\frac{1}{r^2}\,dr$$

$$= \frac{q^2}{8\pi\epsilon_0}\lim_{a\to 0}\left[-\frac{1}{r}\right]_a^\infty$$

$$\lim_{a\to 0}\frac{q^2}{8\pi\epsilon_0 a}$$



Potentials of a point charge



Point charge w/ posn $\vec{w}(t)$.
You sit at posn $\vec{r}$.

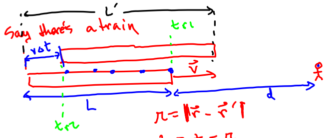
$$V(\vec{r},t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,',t_r)}{\mathscr{r}}\,d\tau'$$

$$\rho(\vec{r}\,',t_r) = q\,\delta^3(\vec{w}(t_r)-\vec{r}\,')$$

$$V(\vec{r},t) = \frac{1}{4\pi\epsilon_0}\int \frac{q\,\delta^3(\vec{w}(t_r)-\vec{r}\,')}{\mathscr{r}}\,d\tau'$$

$$V(\vec{r},t) \neq \frac{1}{4\pi\epsilon_0}\frac{q}{\mathscr{r}}$$

Say there's a train



$$\mathscr{r} = |\vec{r} - \vec{r}\,'|$$

$$t_r = t - \frac{\mathscr{r}}{c}$$

$$t_{r1} = -\frac{d}{c} \quad \Big\{ \text{to see it at } t = \emptyset$$

$$t_{r2} = -\frac{d - L + v\Delta t}{c}$$

$$\Delta t = \frac{L - v\Delta t}{c}$$

$$c\Delta t = L - v\Delta t$$

$$\Delta t = \frac{L}{c+v}$$

$$t_{r2} = -\frac{d - L - v\left(\frac{L}{c+v}\right)}{c}$$

$$L' - L = v\Delta t$$

$$\Delta t = \frac{L'-L}{v} = \frac{L'}{c}$$

$$\frac{L'}{v} - \frac{L}{v} = \frac{L'}{c}$$

$$L'(c-v) = Lc$$

$$L' = \frac{L}{1 - v/c} \quad \Big\{ \text{train looks longer.}$$

Volume of the train looks like $\tau' = \dfrac{\tau}{1 - v/c}$

From behind the opposite is true $\tau' = \dfrac{\tau}{1 + v/c}$

From the side (perpendicular to $\vec{v}$)

$$\tau' = \tau$$

Generalizing you get

$$\tau' = \frac{\tau}{1 - \hat{\mathscr{r}}\cdot\vec{v}/c}$$

$$\int \rho(\vec{r}', t_r)\, d\tau$$

for a point charge is

$$\frac{q}{1 - \hat{\mathscr{r}} \cdot \vec{v}/c}$$

$$V(\vec{r}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}', t_r)}{\mathscr{r}}\, d\tau'$$

$$= \frac{1}{4\pi\epsilon_0} \frac{1}{\mathscr{r}} \frac{q}{1 - \hat{\mathscr{r}} \cdot \vec{v}/c}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q}{\mathscr{r} - \vec{\mathscr{r}} \cdot \vec{v}/c}$$

$$\vec{A}(\vec{r}, t) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}', t_r)}{\mathscr{r}}\, d\tau'$$

$$\vec{J} = \rho \vec{v}$$

$\vec{v}$ evaluated at t_r

$$= \frac{\mu_0}{4\pi} \int \frac{\rho(\vec{r}', t_r)\vec{v}(t_r)}{\mathscr{r}}\, d\tau'$$

$$= \frac{\mu_0}{4\pi} \frac{1}{\mathscr{r}} \frac{q}{1 - \hat{\mathscr{r}} \cdot \vec{v}/c} \vec{v}$$

$$= \frac{\mu_0 \epsilon_0}{4\pi\epsilon_0} \frac{q}{\mathscr{r} - \vec{\mathscr{r}} \cdot \vec{v}/c} \vec{v}$$

$$\vec{A} = \frac{\vec{v}}{c^2} V(\vec{r}, t)$$

$$\vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t}$$

acceleration

$$\vec{E}(\vec{r},t) = \frac{q}{4\pi\epsilon_0} \frac{\mathscr{r}}{(\vec{\mathscr{r}}\cdot\vec{u})^3}\left[(c^2-v^2)\vec{u} + \vec{\mathscr{r}}\times(\vec{u}\times\vec{a})\right]$$

$\vec{v}, \vec{a}, \vec{u}$ all evaluated at t_r.

$$\vec{u} \equiv c\hat{\mathscr{r}} - \vec{v}$$

$$\vec{B}(\vec{r},t) = \frac{1}{c}\hat{\mathscr{r}} \times \vec{E}(\vec{r},t)$$

goes like $\propto \frac{1}{\mathscr{r}^2}$ as you get far away

goes like $\propto \frac{1}{\mathscr{r}}$ as you get far away.

$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$, if E, B both go $\propto \frac{1}{r}$

Power out $= \oint \vec{S}\cdot d\vec{a}$

surface enclosing charge

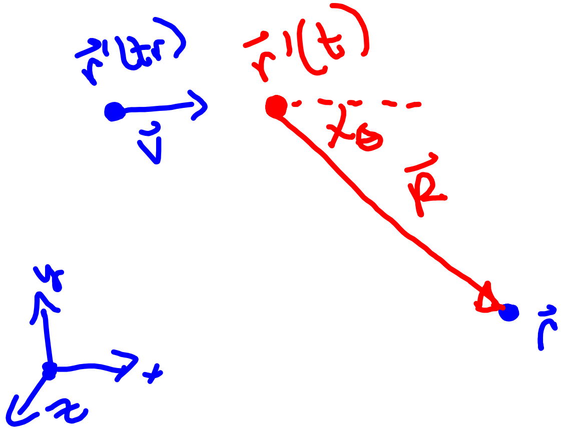
Take a very big sphere of radius r.

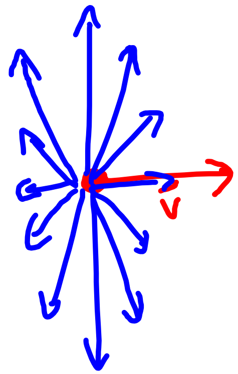
$$\lim_{r\to\infty} \int \vec{S}\cdot\hat{r}\, r^2 \sin\theta\, d\theta\, d\phi$$

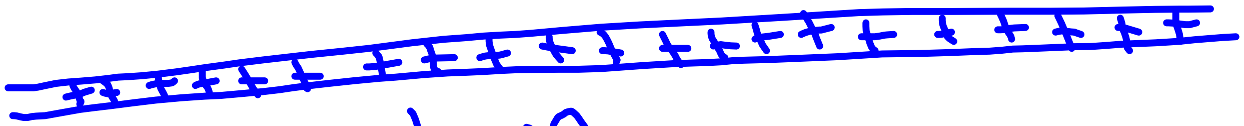
If $\vec{E}, \vec{B}$ go like $\frac{1}{\mathscr{r}} \rightarrow \vec{S}$ goes like $\frac{1}{\mathscr{r}^2}$ and the integral above doesn't go to zero.

$\vec{E}$ field of a pt. charge moving at constant $\vec{v}$.

$$\vec{E}(\vec{r},t) = \frac{q}{4\pi\epsilon_0}\,\frac{1-v^2/c^2}{\left[1-\frac{v^2}{c^2}\sin^2\theta\right]^{3/2}}\,\frac{\hat{R}}{R^2}$$







line charge ρ

Make that go speed c to the right, do $\vec{E}$, $\vec{B}$ go to infinity