Intro to addition of angular momentum
+ spectroscopic notation

spin 1/2: 2 electrons
how many ways can we combine spins?
$|+\rangle|+\rangle \quad |+\rangle|-\rangle \quad |-\rangle|+\rangle \quad |-\rangle|-\rangle$

- in this representation, each spin is marked individually
  - can identify which electron is + or −
- we can ask: what is total spin?

$\vec{S} = \vec{S}_1 + \vec{S}_2$

as with general angular mom, total spin can be described with 2 quantum numbers
S, S_z

range of allowed S_z: $-S \ldots +S$ w/integer spacing
S total: 0 or 1

$|S, S_z\rangle = |1, 1\rangle = |+\rangle|+\rangle$ ↑↑
spins are parallel, both up
$|1, -1\rangle = |-\rangle|-\rangle$ ↓↓
two remaining:
$|1, 0\rangle$ and $|0, 0\rangle$

☆ we can't distinguish btw particles
$\rightarrow |1, 0\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle|-\rangle + |-\rangle|+\rangle\right)$ symmetric on exchange

$|0, 0\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle|-\rangle - |-\rangle|+\rangle\right)$ antisymm.

$|0, 0\rangle$ = "singlet state"
$|1, S_z\rangle$ = "triplet state"

general case:

$\vec{J} = \vec{L}_1 + \vec{L}_2$ $\qquad$ J = total

$\vec{L}_i$ = orbital or spin.

allowed ranges

$j = |l_1 - l_2| \cdots |l_1 + l_2|$ integer spacing.

always > 0

$j_z = -j \to +j$ integer spacing.

examples:

$l_1 = 1 \quad l_2 = 1/2$

j	j_z
$1/2$	$\pm\frac{1}{2}$
$3/2$	$\pm\frac{3}{2}, \pm\frac{1}{2}$

$l_1 = 2 \quad l_2 = 1$

j	j_z
1	$0, \pm 1$
2	$0, \pm 1, \pm 2$
3	$0, \pm 1, \pm 2, \pm 3$

each one of these states is a combination of individual $|l_1, l_{1z}\rangle |l_2, l_{2z}\rangle$ states.

- coeff: Clebsch-Gordon coeff.

How does spin state affect energy?

- electrons have spatial $\psi(\vec{r})$
- symmetric combinations: e^- on avg closer → higher energy.

Ex He $1s^2$ ground state

each e^- in same spatial ψ: S

∴ spin state must be antisymmetric

→ singlet → ^1S

1st excited state $1s2s$

can make symm or antisymm combinations of spatial ψ:

$$\psi(r_1, r_2) = \frac{1}{\sqrt{2}}\left(\psi_{10}(\vec{r}_1)\,\psi_{20}(\vec{r}_2) \pm \psi_{10}(\vec{r}_2)\,\psi_{20}(\vec{r}_1)\right)$$

overall state must be antisymm.

$\therefore$ spin state can be singlet or triplet:

^1S $\quad$ $^3S \rightarrow$ antisymm spatial

$\therefore$ lower energy.