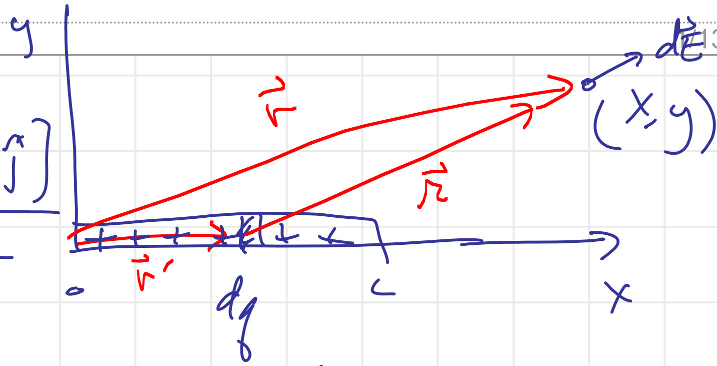
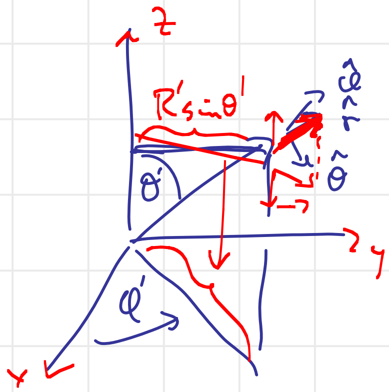
Lecture 2

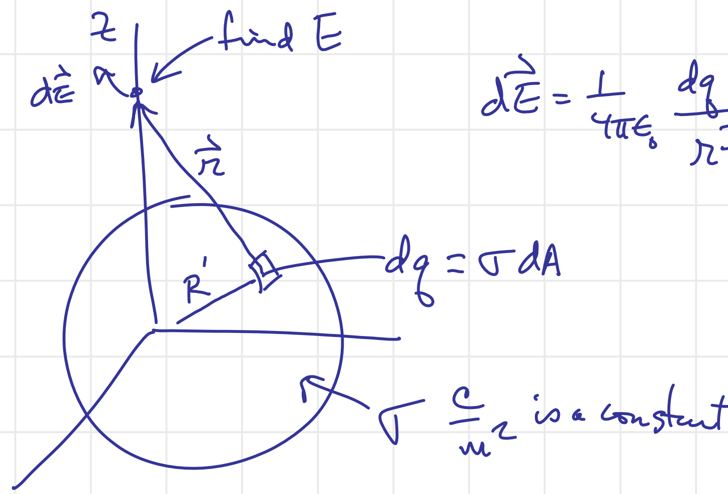
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int_0^L \frac{\lambda\, dx'\left[(x-x')\hat{i}+y\hat{j}\right]}{\left[(x-x')^2+y^2\right]^{3/2}}$$



$\vec{E}$ for a pt charge when $x \gg x'$ $(0 \to L)$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int_0^L \frac{\lambda\, dx'\left[x\left(1-\frac{x'}{x}\right)\hat{i}+y\hat{j}\right]}{\left[x^2\left(1-\frac{x'}{x}\right)^2+y^2\right]^{3/2}} \qquad \delta = \frac{x'}{x} \text{ let } \delta \to 0$$





$$d\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{dq\,\vec{\mathfrak{r}}}{\mathfrak{r}^3}$$

$$\vec{\mathfrak{r}} = \vec{r} - \vec{r}\,'$$

$$\vec{r} = 0\hat{i} + 0\hat{j} + z\hat{k}$$

$$\vec{r}\,' = R'\hat{r}' = R'\hat{r}$$

$$\vec{r}\,' = R'\sin\theta'\cos\varphi'\,\hat{i} + R'\sin\theta'\sin\varphi'\,\hat{j} + R'\cos\theta'\,\hat{k}$$

$$\vec{\mathfrak{r}} = \vec{r} - \vec{r}\,' = -R'\sin\theta'\left(\cos\varphi'\,\hat{i} + \sin\varphi'\,\hat{j}\right) + \left(z - R'\cos\theta'\right)\hat{k}$$

$$d\vec{E} = k\frac{dq}{\mathfrak{r}^3}\vec{\mathfrak{r}} = k\frac{dq}{\mathfrak{r}^2}\hat{\mathfrak{r}}$$

$$|\vec{\mathscr{r}}| = \Big[R'^2 \sin^2\theta' \underbrace{(\cos^2\phi' + \sin^2\phi')}_{1} + \underbrace{(z - R'\cos\theta')^2}_{z^2 - 2zR'\cos\theta' + R'^2\cos^2\theta'}\Big]^{1/2}$$

$$|\vec{\mathscr{r}}| = \Big[R'^2 \sin^2\theta' + z^2 - 2zR'\cos\theta' + R'^2\cos^2\theta'\Big]^{1/2}$$

$$|\vec{\mathscr{r}}| = \Big[R'^2 + z^2 - 2zR'\cos\theta'\Big]^{1/2}$$

flux : $\int \vec{j}\cdot d\vec{a}$

$$\vec{j} = \rho\,\vec{v}$$

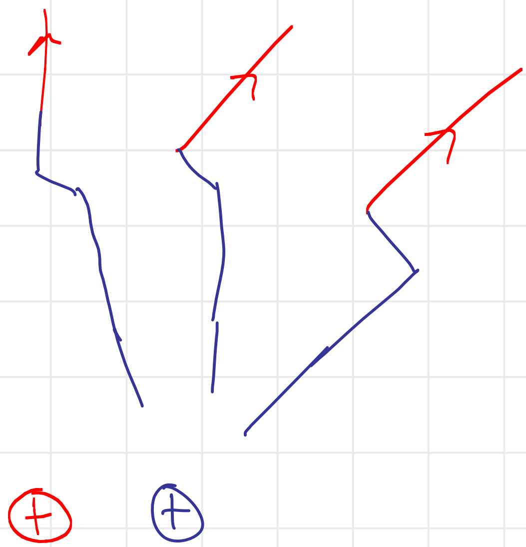
$\vec{v}$: $\frac{m}{s}$ $\qquad$ ρ: $\frac{\text{Coul}}{m^3}$ $\qquad$ $\vec{j}$: $\frac{\text{Coul}}{m^2\,s}$

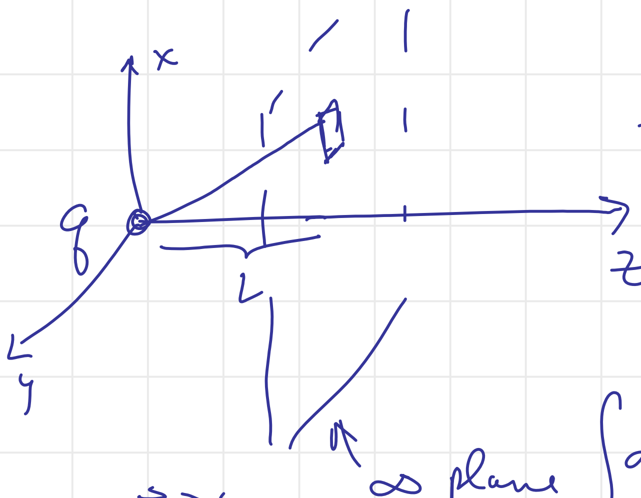
$\int \vec{E}\cdot d\vec{a}$

open surface

$\oint \vec{E}\cdot d\vec{a}$

closed





find flux thru ∞ plane

$$\int \vec{E}\cdot d\vec{a}, \quad d\vec{a} = dx\,dy\,\hat{k}$$

$$\int d\vec{E} = \int k\frac{dq}{r^2}\hat{r} \to \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dy\,dx \text{ - - - - - -}$$

$$\vec{r} = \vec{r} - \underset{0}{\vec{r}'} = x\hat{i} + y\hat{j} + L\hat{k}$$

$$|\vec{r}| = \sqrt{x^2+y^2+L^2}$$