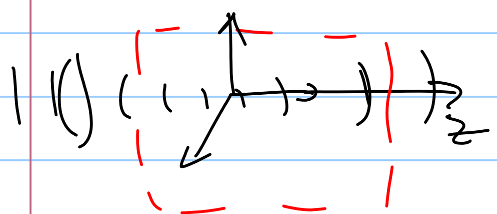
Note Title 8/1/2006

$$\nabla^2 \vec{E} = \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}$$



traveling wave

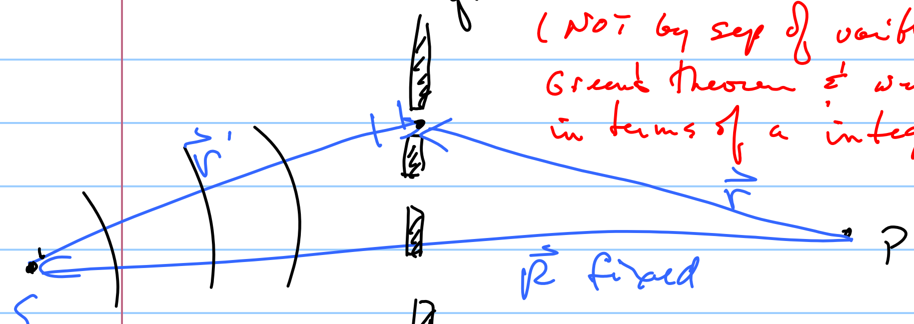
$$\vec{E}(x,y,z,t) \simeq \psi(x,y,z)\, e^{i(kz-\omega t)}$$

scalar function that varies slowly with z

(works sound waves . . .)

Diffraction: solution of the PDE associated with wave eqn (scalar wave eqn)

(NOT by sep of variables by starting with Green's theorem & writing the soln in terms of a integral on surface (aperture))



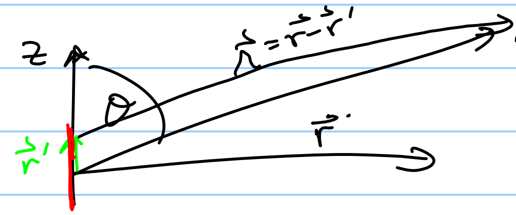
ψ or E

$$U(P) = -\frac{ikU_0 e^{-i\omega t}}{4\pi}\int \frac{e^{ik(r+r')}}{r r'}\left[\cos(\hat{n},\vec{r}) - \cos(\hat{n},\vec{r}\,')\right] \times da$$

$e^{ikr'}$ $\qquad \dfrac{e^{i(kr-\omega t)}}{r}$

firing of wavelets

Antenna:



$\lambda\nu = c$

$\omega = \frac{c}{\lambda} 2\pi = ck$

$\frac{\omega r}{c} = kr$

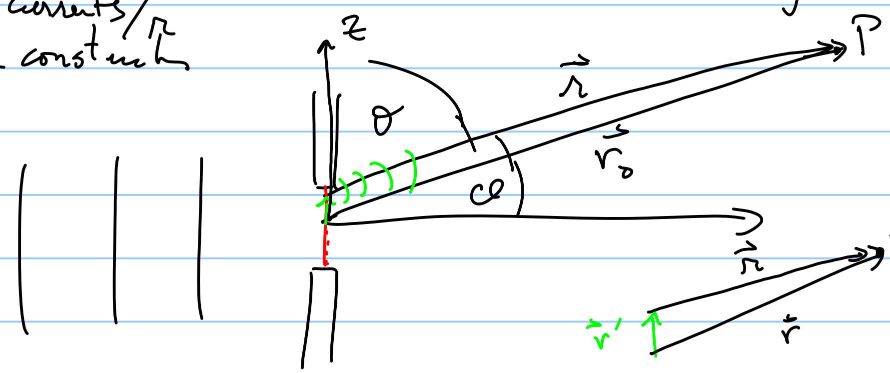
$e^{i(kr-\omega t)}$

$\vec{J}(\vec{r}\,')\, e^{-i\omega t}$

$$\vec{A}(\vec{r}, t) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}\,', t_r)}{r}\, d\tau' = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}\,')\, e^{-i\omega(t - \frac{r}{c})}}{r}\, d\tau'$$

Sum retarded currents/r

Huygens construction



$$U(P) = -\frac{ikU_0}{4\pi} e^{-i\omega t} \int \frac{e^{ik(r'+r)}}{r r'} \left[\cos(\hat{n}, \vec{r}) - \cos(\hat{n}, r')\right] \times da$$

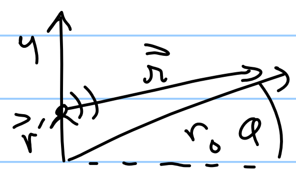
$$= \text{Const} \int \frac{e^{i(k\mathcal{R} - \omega t)}}{\mathcal{R}} \, da \qquad r \gg r'$$

$$|\vec{\mathcal{R}}| = \sqrt{\vec{\mathcal{R}}\cdot\vec{\mathcal{R}}} = \sqrt{(\vec r - \vec r\,')\cdot(\vec r - \vec r\,')} = \sqrt{r^2 + r'^2 - 2\vec r\cdot\vec r\,'}$$

$$\simeq r\left(1 - \frac{r'}{r}\cos\theta\right)$$

$\cos(90-\varphi) = \sin\varphi$

Diffraction



$$\mathcal{R} \simeq r_0\left(1 - \frac{r'}{r_0}\cos\theta\right)$$

$$= r_0 - r'\sin\varphi$$

$$= r_0 - y\sin\varphi$$

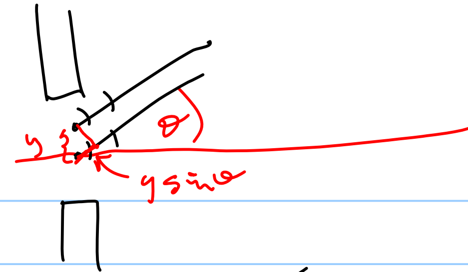
θ before

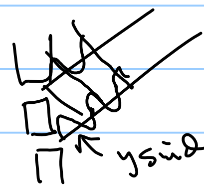
diff

$$e^{ik\mathcal{R}} \simeq e^{ik(r_0 - y\sin\theta)}$$

Sum retarded amplitudes in the integral

$$U \propto e^{ikr_0}\int e^{iky\sin\theta}\, dy$$

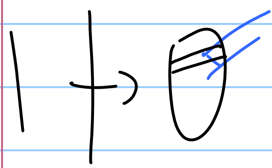


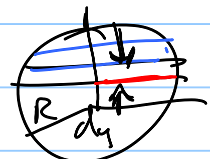


y sinθ = mλ const

$x^2+y^2=R^2$ $\quad x=\sqrt{R^2-y^2}$

$da = 2\sqrt{R^2-y^2}\,dy$

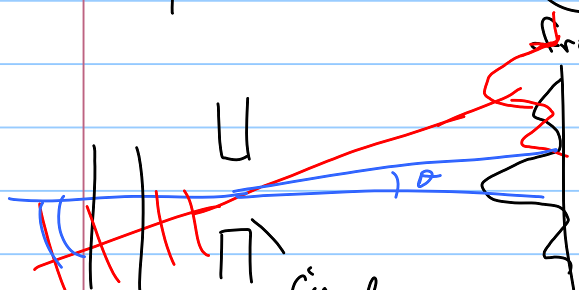




front view

$U \propto \int e^{iky\sin\theta}\, da$

$da = 2\sqrt{R^2-y^2}\,dy$



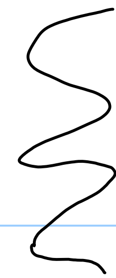
Circular apun

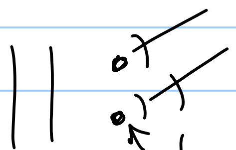
front view

i

Crystallography requires $\lambda < d$

cons int $\lambda = d \sin\theta$

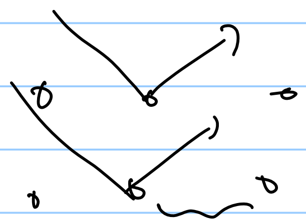




atom in lattice

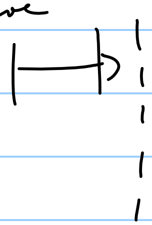
$\sin\theta = \frac{\lambda}{d} = \frac{6000\,\text{Å}}{1\,\text{Å}} = 6000$

to see diffraction effect $\lambda < d$



d

microwave



$d < \lambda = \frac{c}{\nu} = \frac{3\times10^{8}}{3\times10^{9}} = .1\text{ m}$

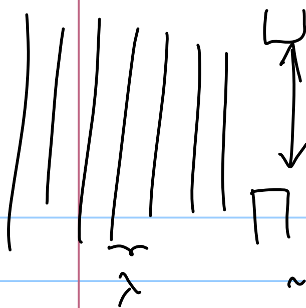
??

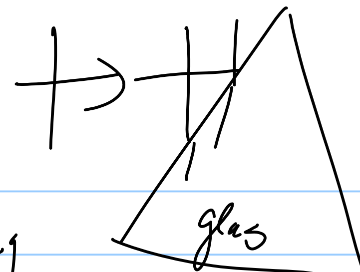
10^{-3} m

λ visible $\approx .6\times10^{-6}$ m $<< d = 10^{-3}$

geometrical & physical optics

apertures are $> \lambda$



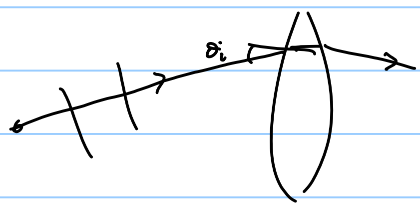


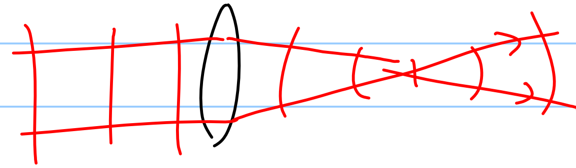
~ no bending
waves travel in straight lines

rays $\perp$ wave fronts

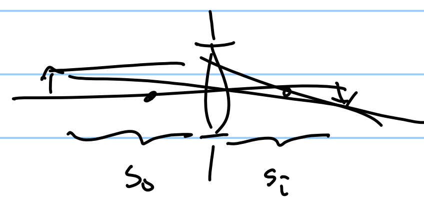
physics: $\angle$ incid = $\angle$ reflection

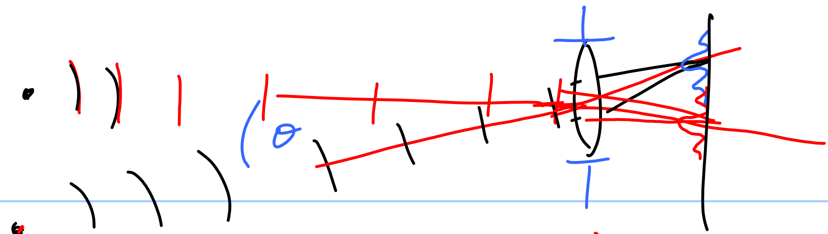
snells law $n_i \sin\theta_i = n_r \sin\theta_r$



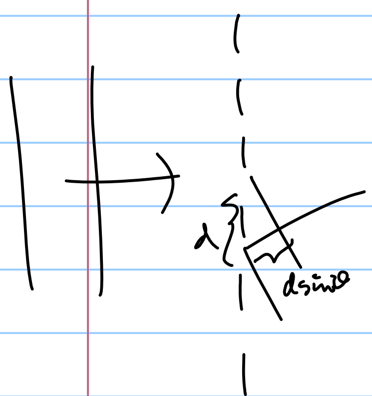


$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$



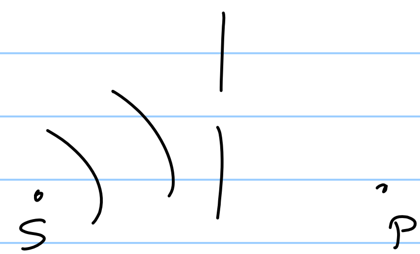


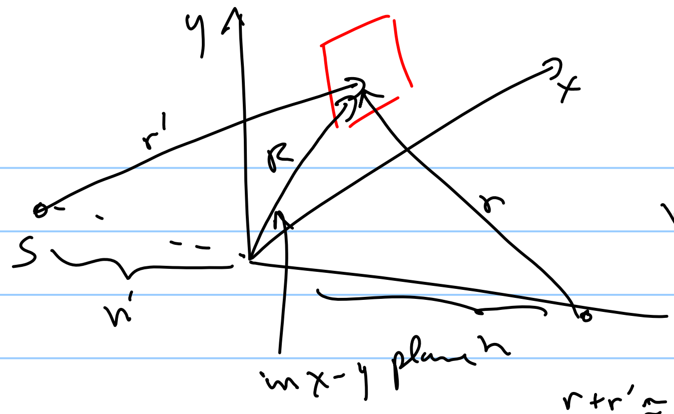
$d \gg \lambda$ so geometrical optics
should be ok, diff still occurs
it spreads the image



$$\int_a^b e^{iky\sin\theta}\,dy + \int_c^d \text{—} + \cdots$$

Fresnel diffraction





$h > R$

$R^2 = x^2 + y^2$

$r + r' = \sqrt{R^2 + h^2} + \sqrt{R^2 + h'^2}$

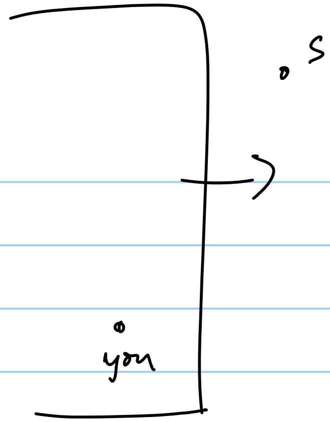
$r + r' = h\sqrt{1 + \frac{R^2}{h^2}} + h'\sqrt{1 + \frac{R^2}{h'^2}}$

$r + r' \simeq h + h' + \frac{1}{2}R^2\left(\frac{1}{h^2} + \frac{1}{h'^2}\right)$

$$U(P) = -\frac{ikU_0 e^{-i\omega t}}{4\pi}\int \frac{e^{ik(r'+r)}}{rr'}\underbrace{\left[\cos(\hat{n},\hat{r}) - \cos(\hat{n},r')\right]}_{\sim\text{ const}} \times da$$

$r + r' = h + h' + \frac{1}{2L}(x^2 + y^2)$ $\qquad L = \frac{1}{\left(\frac{1}{h} + \frac{1}{h'}\right)}$

$$U(p) = \text{Const} \int_{x_1}^{x_2}\int_{y_1}^{y_2} e^{ik\frac{1}{2L}(x^2+y^2)}\,dx\,dy$$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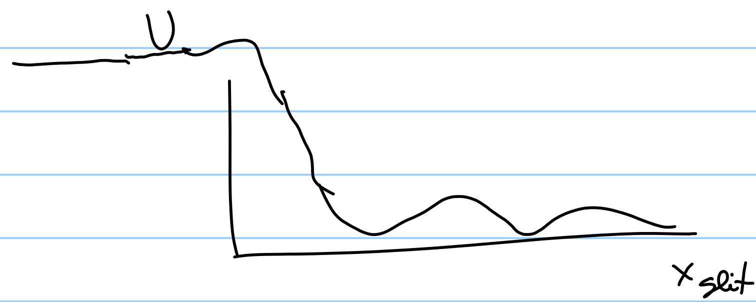
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x_{slit}