PHGN361 2010 Practice Exam 2: NAME

Start from fundamental principles and derive all results. **Explain each step for credit.**

1. Derive an expression for (a) the potential and (b) the electric field inside a parallel plate capacitor using separation of variables. The plates are separated by a distance d along the z axis. Assume infinite plates and a 12 V battery across the plates with the lower plate grounded. Explain how you would check your answer.

$V(x,y,z) = X(x)\,Y(y)\,Z(z)$ put into $\nabla^2 V = 0 = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$

$\Rightarrow YZ\frac{d^2X}{dx^2} + XZ\frac{d^2Y}{dy^2} + XY\frac{d^2Z}{dz^2} = 0$

$Y(y)$ cannot depend on y since changing the y position yields the same situation in an infinite capacitor $\Rightarrow C_2 = 0$

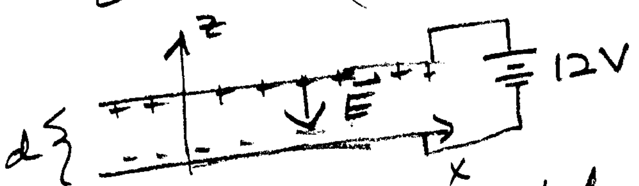
$X(x)$ cannot depend on x for the same reason $\Rightarrow C_1 = 0$

Since $\underset{0}{C_1} + \underset{0}{C_2} + C_3 = 0 \Rightarrow C_3 = 0$. The ODE is then $\frac{d^2Z}{dz^2} = 0$

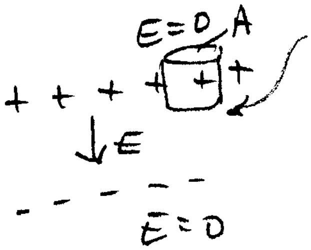
$\Rightarrow Z(z) = mz + b$ with boundary conditions $Z(z=0) = 0$, $Z(z=d) = 12$

This is satisfied by $Z(z) = \frac{12}{d}z + 0$

$\vec{E} = -\vec{\nabla}V = \left(-\hat{x}\frac{\partial}{\partial x} - \hat{y}\frac{\partial}{\partial y} - \hat{z}\frac{\partial}{\partial z}\right)\left(\frac{12}{d}z\right) = -\hat{z}\frac{12}{d}$ constant $\vec{E}$



$\oint \vec{E}\cdot d\vec{a} = \underbrace{\int_{caps}}_{EA} + \underbrace{\int_{body}}_{\vec{E}\perp d\vec{a}} \overset{0}{}$



$Q_{encl} = \sigma A$

check using Gauss's law which yields $\vec{E} = -\hat{z}\frac{\sigma}{\epsilon_0}$

$EA = \frac{\sigma A}{\epsilon_0} \Rightarrow E = \frac{\sigma}{\epsilon_0}$

$-\int_0^d \vec{E}\cdot d\vec{l} = +\underbrace{\int_0^d E\,dz}_{\frac{\sigma d}{\epsilon_0}} = \underbrace{V(z=d) - V(z=0)}_{12V} \Rightarrow \sigma = \frac{12\epsilon_0}{d}$

So $\vec{E} = -\hat{z}\frac{12\epsilon_0}{\epsilon_0 d} = -\hat{z}\frac{12}{d}$ checks