Vector potential

$$\begin{tikzcd} & J \arrow[dl] \arrow[dr] & \\ \vec{A} \arrow[rr, "\vec{B} = \vec{\nabla}\times\vec{A}"'] & & \vec{B} \end{tikzcd}$$

Max Eqn B

$$\vec{\nabla}\times\vec{B} = \mu_0 \vec{J} \qquad \vec{\nabla}\cdot\vec{B} = 0$$

cons charge

$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t} = \phi$$

Magneto-statics

Guess

$$\vec{B} = -\vec{\nabla} v \qquad \vec{\nabla}\times\vec{B} = -\vec{\nabla}\times\vec{\nabla} v \overset{\text{always}}{\equiv} 0$$

$$\vec{E} = -\vec{\nabla} V$$

prove

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial x}v & \frac{\partial}{\partial y}v & \frac{\partial}{\partial z}v \end{vmatrix}$$

Guess $\vec{B} = \vec{\nabla}\times\vec{A}$

$$\vec{\nabla}\cdot\vec{B} = \vec{\nabla}\cdot\vec{\nabla}\times\vec{A} \overset{\text{always}}{\equiv} 0$$

$$\vec{\nabla}\times\vec{B} = \vec{\nabla}\times\vec{\nabla}\times\vec{A} = \mu_0 \vec{J}$$

$$\vec{B} = \vec{\nabla} \times \vec{A} \qquad \vec{E} = -\vec{\nabla} V$$

$V' + \text{const}$

$$\vec{A} \to \vec{A}' + \vec{\nabla}\psi$$

scalar function

$f(x, y, z)$

$$\vec{\nabla} \times (\vec{A} + \vec{\nabla}\psi) = \vec{\nabla} \times \vec{A} + \underbrace{\vec{\nabla} \times \vec{\nabla}\psi}_{\substack{\shortparallel \\ 0}}$$

always

Ex

$$\vec{A} = -\frac{B_0 y}{2}\hat{x} + B_0\frac{x}{2}\hat{y} + \phi\hat{z}$$

$$\vec{A}' = \vec{A} - \vec{\nabla}\psi = -B_0 \frac{y}{2}\hat{x} + B_0\frac{x}{2}\hat{y} - \frac{B_0 y}{2}\hat{x} - \frac{B_0 x}{2}\hat{y}$$

$\psi = B_0\left(\frac{xy}{2}\right)$

$$= -B_0 y\hat{x}$$

$$\vec{A}' = -B_0 y\hat{x}$$

What is $\vec{B}$ for these two A's

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -\frac{B_0 y}{2} & \frac{B_0 x}{2} & 0 \end{vmatrix} = 0\hat{x} + 0\hat{y} + \left(\frac{B_0}{2} + \frac{B_0}{2}\right)\hat{z}$$

$-B_0 y$ $\qquad = B_0\hat{z}$

$\vec{B} = \vec{\nabla} \times \vec{A}$ 1 eqn for $\vec{A}$

Need two eqn to specify $\vec{A}$

(1) $\vec{\nabla} \times \vec{A}$

(2) $\vec{\nabla} \cdot \vec{A}$ $\vec{A} \to \vec{A}' + \vec{\nabla}\psi$

$$\vec{\nabla} \cdot \vec{A} = \vec{\nabla} \cdot \vec{A}' + \underbrace{\vec{\nabla} \cdot \vec{\nabla}\psi}_{\nabla^2\psi}$$

by a choice ψ we can make $\vec{\nabla} \cdot \vec{A}$ anything

Magnetostatic choose $\psi \Rightarrow \nabla^2\psi + \vec{\nabla} \cdot \vec{A}' = \phi$

$\vec{\nabla} \cdot \vec{A} = 0$ $\vec{B} = \vec{\nabla} \times \vec{A}$

Coulomb guage

$$\vec{\nabla} \times \vec{B} = \underbrace{\vec{\nabla} \times \vec{\nabla} \times \vec{A}} = \mu_0 \vec{J}$$

$$\vec{\nabla}\left(\cancel{\vec{\nabla} \cdot \vec{A}}\right) - \nabla^2\vec{A} = \mu_0 \vec{J}$$

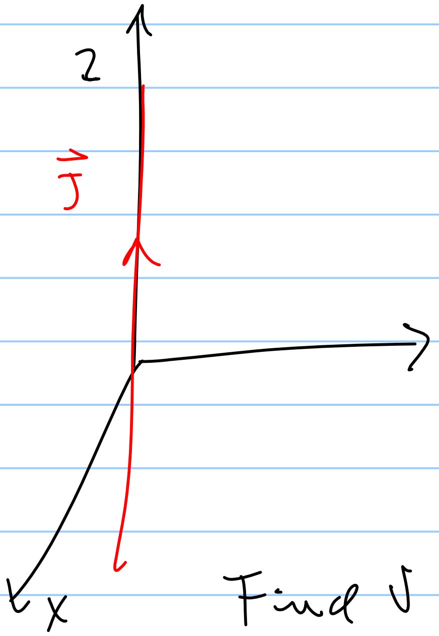
0

$$\frac{\partial^2}{\partial x^2} A_x \hat{x} + \frac{\partial^2}{\partial y^2} A_y \hat{y} + \frac{\partial^2}{\partial z^2} A_z \hat{z}$$

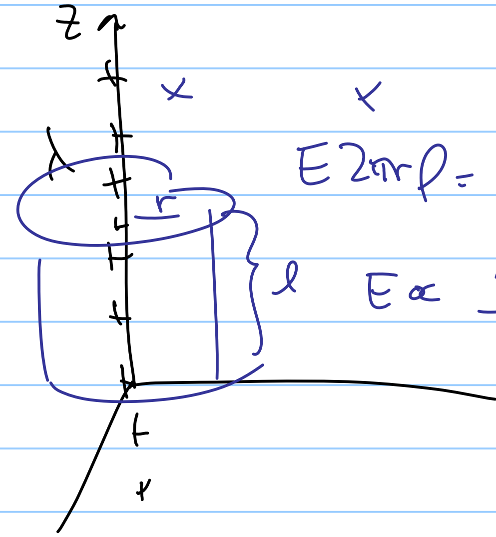
direction of A is the direction of J

$\nabla^2 A_x = -\mu_0 J_x \qquad \nabla^2 A_y = -\mu_0 J_y \qquad \nabla^2 A_z = -\mu_0 J_z$

LIKE $\qquad \nabla^2 V = -\rho/\epsilon_0$



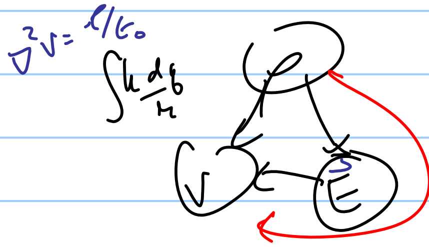
$\approx$



$E\, 2\pi r \ell = \lambda \ell$

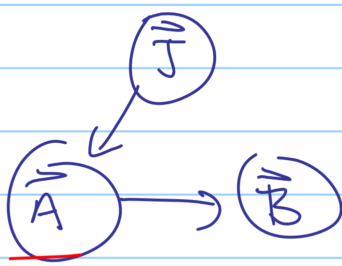
$E \propto \frac{1}{r}$

Find V: 1^{st} find E from Gauss



$2^{nd} \quad \Delta V = -\int \vec{E}\cdot d\vec{\ell}$

$\int \frac{1}{r}\, dr \to \ln(r)$



$\vec{A} \propto \ln(r)\, \hat{z}$

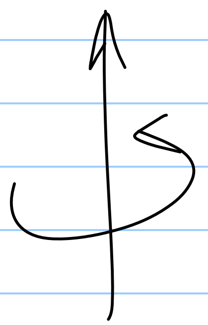
$$\vec{B} = \vec{\nabla} \times \vec{A} \propto \text{const} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \phi & \phi & \ln\sqrt{x^2+y^2} \end{vmatrix}$$

$$B_x \propto \frac{\partial}{\partial y} \ln\sqrt{x^2+y^2} \propto \frac{1}{\sqrt{x^2+y^2}} \frac{\frac{1}{2}2y}{\sqrt{}} = \frac{1}{r^2} y$$

$$\frac{d}{dy}(\ )^{1/2} = -\frac{1}{2}\frac{1}{(\ )^{1/2}} 2y$$

$$B_y \propto \frac{x}{r^2}$$

$$|\vec{B}| = \sqrt{B_x^2 + B_y^2} \propto \sqrt{\frac{x^2+y^2}{r^4}} \propto \frac{1}{r^2} r \propto \frac{1}{r}$$

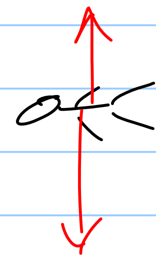


$$B\, 2\pi r = \mu_0 I_{encl}$$

$$B \propto \frac{1}{r}$$

Ohms Law

Skydiver



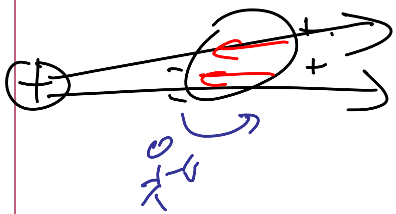
$F_f = b v$

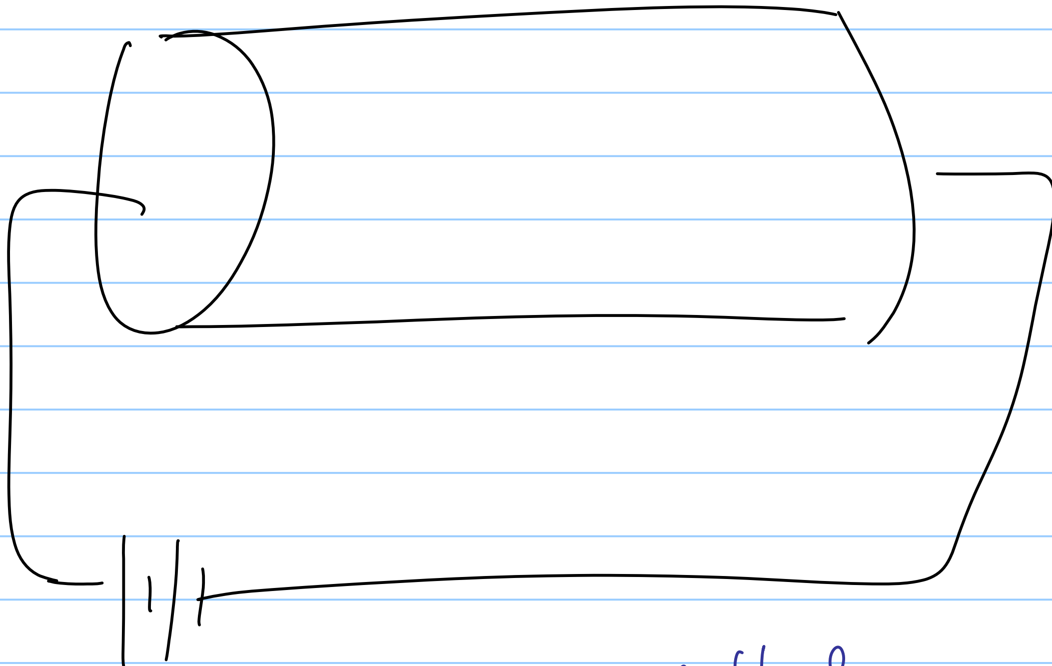
area, density air, - -

mg

$$\sum F = ma = \emptyset \qquad mg - bv = 0$$

$$v = \frac{mg}{b}$$





drift vel

Ohms Law $\qquad v \propto E$

$$\boxed{\vec{J} = \rho v = \sigma \vec{E}}$$

const

Ohms Law

$$\vec{\nabla} \cdot \vec{E} = \vec{\nabla} \cdot \frac{\vec{J}}{\sigma} = \frac{1}{\sigma} \vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

const

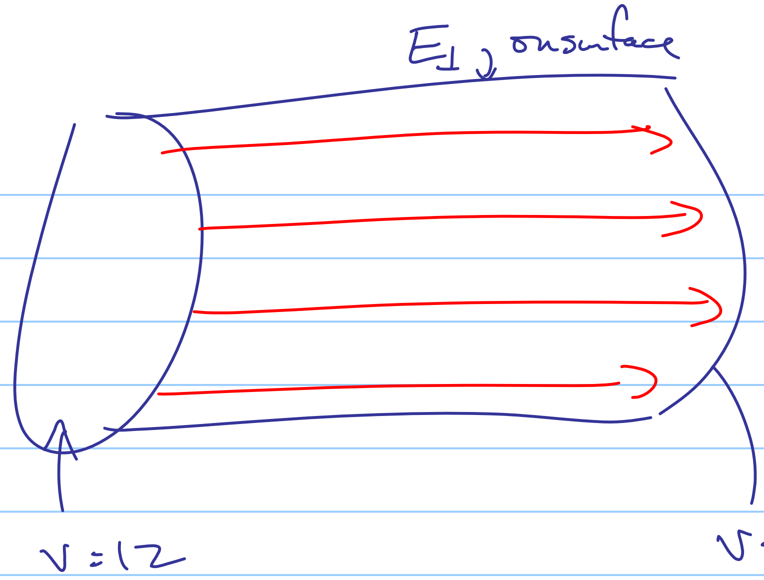
in ohmic material

steady state $= 0$

$$\vec{E} = -\vec{\nabla} V$$

$$\vec{\nabla} \cdot \vec{E} = \nabla^2 V = 0$$

LAPLACES EQN !!



$J_\perp = 0$ Ohms

$J \propto \sigma E$

$J_\perp$ on surface says

$E_\perp = 0$