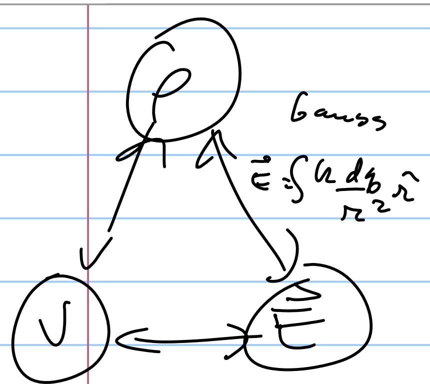
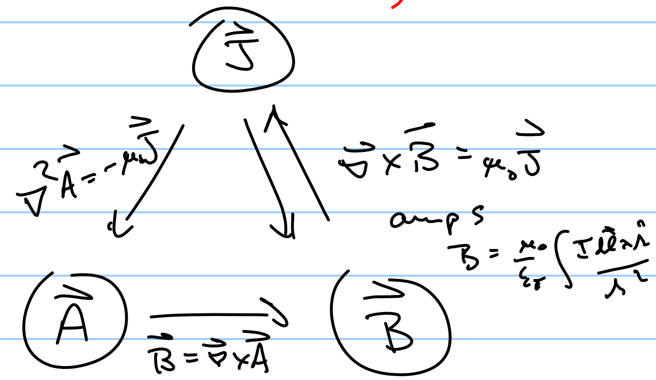
Lecture 29

Do 5.31 (a) only

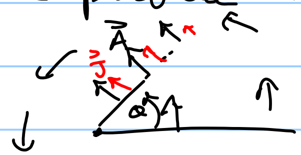
Note Title 4/3/2006





$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

Lecture problem 1



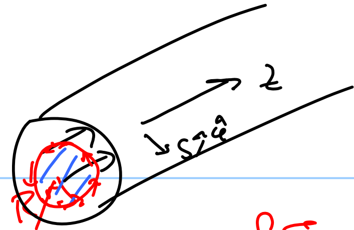
$$\vec{A} = A_\varphi \hat{\varphi} + \underset{0}{A_s} \hat{s} + \underset{0}{A_z} \hat{z}$$

$$A_\varphi = k$$

$$\vec{B} = \nabla \times \vec{A} = \frac{1}{s}\frac{\partial}{\partial s}(s k)\,\hat{z} = \frac{k}{s}\hat{z}$$

$$\vec{J} = \frac{1}{\mu_0}\nabla \times \vec{B} = \frac{1}{\mu_0}\left(-\frac{\partial}{\partial s}\left(\frac{k}{s}\right)\right)\hat{\varphi} = \frac{k}{\mu_0 s^2}\hat{\varphi}$$

5.25 (b)



total current I

J is constant

$$\oint \vec{B}\cdot d\vec{l} = \oint |\vec{B}||d\vec{l}|\cos\underset{0}{\theta} = |\vec{B}|\,2\pi r$$

$$\text{RHS} = I_{enc}\,\mu_0 = \mu_0 \int \vec{J}\cdot d\vec{a} = J\pi r^2$$

(a) $\vec{B} \propto r\,\hat{\phi}$

(b) $\vec{A} = \underset{0}{A_\phi}\,\hat{\phi} + \underset{0}{A_s}\,\hat{s} + \underset{A_z(s)}{A_z}\,\hat{z}$

$$\vec{B} = \vec{\nabla}\times\vec{A} = \left[\frac{1}{s}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right]\hat{s} + \left[\frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s}\right]\hat{\phi} + \frac{1}{s}\left[\frac{\partial}{\partial s}(sA_\phi) - \frac{\partial A_s}{\partial \phi}\right]\hat{z}$$

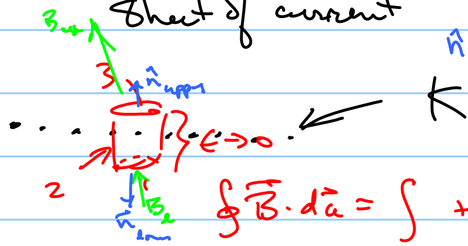
$$-r\hat{\phi} = \frac{\partial A_z}{\partial s}\hat{\phi}$$

$\boxed{\text{Boundary conditions}}$

$$\int \underset{0}{\vec{\nabla}\cdot\vec{B}}\, d\tau = \oint \vec{B}\cdot d\vec{a} = 0$$

Sheet of current

$\hat{n}_{upper} = -\hat{n}_{lower}$

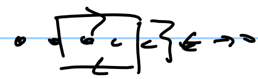


$B_\perp$ across boundary

ε_1 $B_\parallel$ " "

$$\oint \vec{B}\cdot d\vec{a} = \int_1 + \int_2 \quad \int_3$$

$\int |\vec{B}|\, da \cos\theta = -B^{\perp}_{lower} A$ $\qquad \int_2 = B^{\perp}_{upper} A$ $\qquad B^{\perp}_{upper} = B^{\perp}_{lower}$

 $\vec{\nabla}\times\vec{B} = \mu_0 J$

Stokes $\quad \int \vec{\nabla}\times\vec{B}\cdot d\vec{a} = \oint \vec{B}\cdot d\vec{l}$

5.27 (a) $\qquad \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}')}{\mathscr{r}}\, d\tau'$

$$\vec{\nabla}\cdot\vec{A} = \frac{\mu_0}{4\pi}\int \vec{\nabla}\cdot\left(\frac{\vec{J}}{\mathscr{r}}\right) d\tau'$$

operates on unprimed

function of primed

$$\vec{\nabla}\cdot\left(\frac{\vec{J}}{\mathscr{r}}\right) = \frac{1}{\mathscr{r}}\underbrace{\vec{\nabla}\cdot\vec{J}}_{0} + \vec{J}\cdot\vec{\nabla}\left(\frac{1}{\mathscr{r}}\right)$$

$\mathscr{r} = |\vec{r} - \vec{r}'|$ $\qquad \vec{\nabla}\frac{1}{\mathscr{r}} = -\vec{\nabla}'\frac{1}{\mathscr{r}}$

$$\vec{\nabla}\cdot\left(\frac{\vec{J}}{\mathscr{r}}\right) = -\vec{J}\cdot\vec{\nabla}'\left(\frac{1}{\mathscr{r}}\right)$$

Now $\qquad \vec{\nabla}'\cdot\left(\frac{\vec{J}}{\mathscr{r}}\right) = \frac{1}{\mathscr{r}}\underbrace{\vec{\nabla}'\cdot\vec{J}(\vec{r}')}_{0 \text{ magnetostatics}} + \vec{J}\cdot\vec{\nabla}'\frac{1}{\mathscr{r}}$

Apply divergence Theorem: integral over a vol to a surface integral