MATH 225 - Differential Equations
Project 1, Fall 2008

September 14, 2008
Due: September 24, 2008

Lab 1.3 - Logistics Population Models with Harvesting

For this project we consider lab 1.3 of Differential Equations pages 146 to 147. After reading this material construct a lab report addressing each of the following questions for cases 3, 4, 6, 8 of table 1.10 on page 147:[1]

1. Given a logistics growth model with constant harvesting,

$$\frac{dp}{dt} = kp\left(1 - \frac{p}{N}\right) - a, \quad k, N, a \in \mathbb{R}^+. \tag{1}$$

   (a) Construct a list of variables and parameters associated with (1) and describe the meaning of each.

   (b) Analytically solve (1) using the methods discussed in section 1.2 of the text. (for $a = a_1$)

   (c) Discuss qualitative behavior of the solutions to (1) through the equation's: (for $a = a_1$)

      i. Equilibrium Points

      ii. Phase Line

   (d) Using Euler's method and a slope field diagram address the following question:

      - For $a = a_1$, what will happen to the fish population for various initial conditions?

2. Given a logistics growth model with periodic harvesting,

$$\frac{dp}{dt} = kp\left(1 - \frac{p}{N}\right) - a(1 + \sin(bt)), \quad k, N, a, b \in \mathbb{R}^+. \tag{2}$$

   (a) In this case what do the parameters a and b represent?

   (b) Is it possible to solve (2) using the methods discussed in section 1.2 of the text?

   (c) Using Euler's method and a slope field diagram address the following questions:

      i. For $a = a_1$ and $b = 1$ what will happen to the fish population for various initial conditions?

      ii. For $a = a_2$ and $b = 1$ what will happen to the fish population for various initial conditions?

   Explain why there are no equilibrium points and thus no phase line for this problem.

3. Summary and conclusions. In a short essay format summarize your results from the previous questions. Compare and contrast each of the two models. Be sure to justify your conclusions by referencing your previous summary and analysis.

Choice	k	N	a_1	a_2
3	0.2	5	0.21	0.25
4	0.2	5	0.16	0.25
6	0.2	5	0.09	0.25
8	0.2	5	0.24	0.25

[1] Your report should be well organized and clearly presented. If steps are unclear then include more steps or make annotations clarifying the procedure and purpose. Be sure to label and title any included graphs or tables of data.

1(b) Choice 3

$K=0.2$, $N=5$, $a_1=a=0.21$, $a_2=0.25$

$$\int \frac{dp}{0.2p-\frac{0.2}{5}p^2-0.21} = \int dt$$

$$\int \frac{dp}{-0.04p^2+0.2p-0.21} = t+C$$

$$\int \frac{-\frac{1}{0.04}}{(p-1.5)(p-3.5)}\,dp = t+C$$

$$\int\left(\frac{A}{p-1.5}+\frac{B}{p-3.5}\right)dp = t+C$$

$$\int\left(\frac{12.5}{p-1.5}+\frac{(-12.5)}{p-3.5}\right)dp = t+C$$

Implicit Form of Solution:

$$12.5\ln|p-1.5| - 12.5\ln|p-3.5) = t+C$$

$$\ln(|p-1.5|^{12.5}) - \ln(|p-3.5|)^{12.5} = t+C$$

$$e^{\ln\left(\left(\frac{|p-1.5|}{|p-3.5|}\right)^{12.5}\right)} = e^{t+C}$$

$$\left(\frac{p-1.5}{p-3.5}\right)^{12.5} = Ae^{t}, \quad A = \pm e^{C}$$

$$\left(\frac{p-1.5}{p-3.5}\right) = Be^{\frac{t}{12.5}}, \quad B = A^{1/12.5}$$

$$p-1.5 = Be^{t/12.5}p - 3.5Be^{t/12.5}$$

$$p - pBe^{t/12.5} = 1.5 - 3.5Be^{t/12.5}$$

$$p(1-Be^{t/12.5}) = 1.5 - 3.5Be^{t/12.5}$$

Explicit Form:

$$p(t) = \frac{1.5-3.5Be^{t/12.5}}{1-Be^{t/12.5}}$$

* Repeat for other 3 cases

Partial Fractions

$$A(p-3.5)+B(p-1.5) = -\frac{1}{0.04}$$

$$(A+B)p + (-3.5A-1.5B) = -25$$

$(A+B)=0$ and $-3.5A-1.5B=-25$

$B=-A$ and $-2A=-25$

$B=-12.5$ and $A=12.5$

1. (c) Choice 3

$k = 0.2$, $N = 5$, $a_1 = a = 0.21$, $a_2 = 0.25$

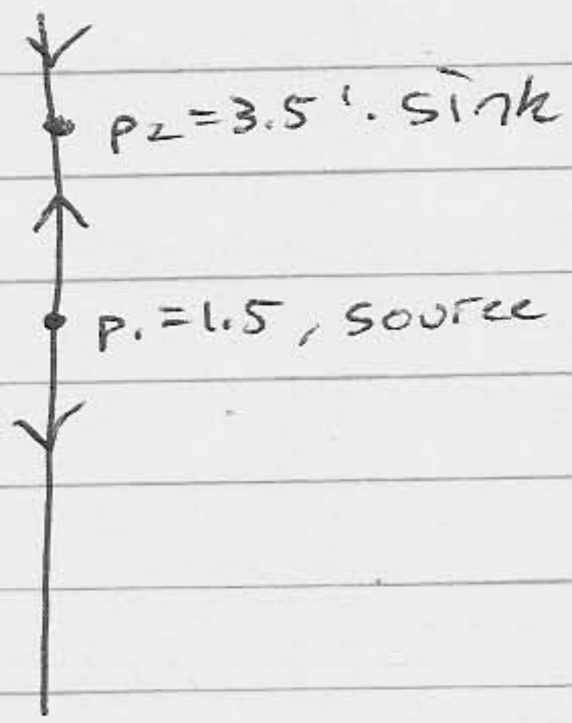
$$\frac{dp}{dt} = f(p) = 0.2p - \frac{0.2}{5}p^2 - 0.21$$

$$= -0.04p^2 + 0.2p - 0.21 = 0$$

* Find roots

$$-0.04(p-1.5)(p-3.5) = 0$$

EP: $p_1 = 1.5$, $p_2 = 3.5$



Linearization Thm

$$f'(p) = -0.08p + 0.2$$

$f'(1.5) = 0.08 > 0$, source

$f'(3.5) = -0.08 < 0$, sink

* Repeat for other 3 cases.